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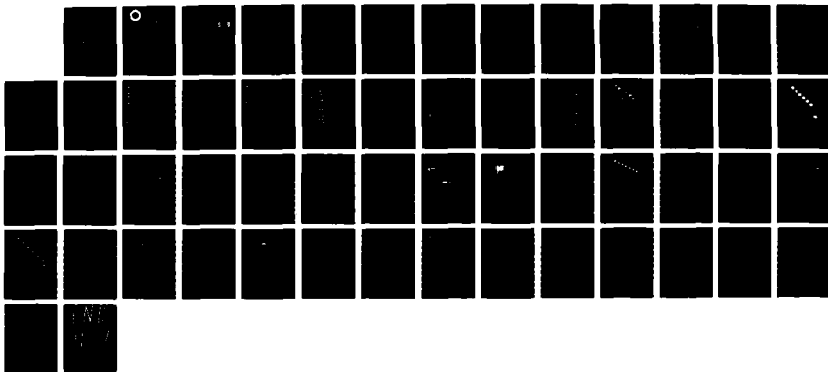
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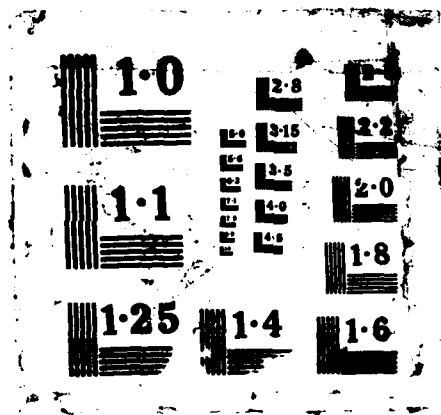
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Test Problems

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Irvin J. Lustig

TECHNICAL REPORT SOL-87-11

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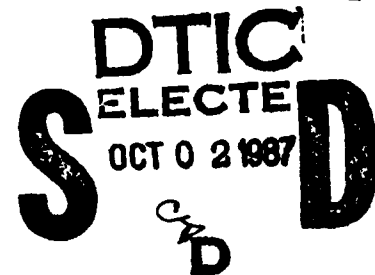
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An Analysis of an Available Set of Linear Programming Test Problems

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Technical Report SOL 87-11

August, 1987

Abstract

A set of linear programming test problems is analyzed with MINOS, Version 5.1. The problems have been run with different options for scaling and partial pricing to illustrate the effects of these options on the performance of the simplex method. The results indicate that the different options can significantly improve or degrade the performance of the simplex method, and that these options must be chosen wisely.

For each problem, a picture of the nonzero structure of the matrix A is also presented so that the problems can be classified according to structure. 

Keywords: Linear programming, Simplex method, Scaling, Partial pricing, Pictures, Computational comparisons

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1. Introduction

The recent development of new algorithms for linear programming has sparked some serious computational testing of these algorithms, with comparisons often being made against the simplex method (Dantzig, 1963), as implemented in some version of MINOS (Murtagh and Saunders, 1980, 1983, 1987). For such tests, MINOS is usually run with certain options set at their default values. Changing the settings of these options affects the performance of the simplex method, and hence can affect the conclusions one makes when comparing the performance of MINOS to another linear programming code. The purpose of this paper is to analyze how much some of these options can change the speed of MINOS when solving linear programs.

Gay (1985) has made a set of linear programming test problems publicly available via *netlib* (Dongarra and Grosse, 1987). The form of each problem is

$$\begin{aligned} \min \quad & c^T x \\ \text{subject to} \quad & Ax \diamond b, \\ & l \leq x \leq u, \end{aligned}$$

where the symbol \diamond indicates that each row a_i^T is related to b_i by a relation of the form $a_i^T x \leq b_i$, $a_i^T x \geq b_i$, or $a_i^T x = b_i$. At the time of this study, 53 problems were in the library. Table 1.1 lists the problems, ordered by the number of nonzeros. Each problem was given a number by Gay, which is reflected in the last column. For ease of presentation of some graphs later in this report, the problems have been renumbered from 1 to 53, as listed in the first column. The optimal objective values, as reported by Gay, are also listed. Later, it will be interesting to note the slight differences between these values and the values obtained by MINOS when the scaling and partial pricing options are varied. Table 1.2 gives some statistics on each problem indicating the number of different types of rows and columns. A normal row or column has either a lower or upper bound. A bounded row or column has distinct lower and upper bounds. A fixed row or column has equal lower and upper bounds, while a free column has no lower or upper bound.

In Section 2, the testing methodology is explained and the results are presented for six different combinations of MINOS options. Section 3 contains an analysis and discussion of these results. Section 4 presents pictures of the nonzero structure of the constraint matrix for each of the problems.

2. Testing Methodology and Results

The computational testing in this report was performed on a DEC VAXstation II with 13 megabytes of main memory. The operating system was VMS, version 4.5, and the

VMS FORTRAN compiler, version 4.6, was used with the default options, including code optimization. Each run was made as a batch job, and the account used to run the tests was given access to all of the main memory. Hence, the memory used for each test was never paged onto the disk, and the amount of paging was kept to a minimum. The VMS timing mechanism used is slightly affected by disk input and output (such as printing the iteration log), and hence all timing results contain a slight but negligible error.

For each test run, the total time of the run and the solution time were recorded. The solution time was measured by timing the MINOS subroutine M5SOLV. MINOS, Version 5.1, (June 1987) was used. In the course of running the experiments, a degenerate cycle of 6 was discovered in one of the runs on the problem PILOTJA, with this cycle occurring in Phase 2 after 43000 total iterations. MINOS was modified to avoid such cycles and all of the problems were run again. The main changes required were in the step-length procedure M5CHZR. These changes will be documented elsewhere and made available in a later version.

MINOS requires a specifications file for each run. The file used in all cases was of the form:

```
BEGIN  SPECS file for timing LP problems
ROWS                2400
COLUMNS            10000
ELEMENTS             45000
SUMMARY FILE         9
MPS FILE              10
ITERATIONS           100000
PRINT LEVEL          0
SCALE OPTION         s
PARTIAL PRICE        p
END
```

The options changed were the scaling option s and the partial pricing option p . The default values are $s = 0$ and $p = 1$. The six combinations used were $s = 0$ with $p = 1$ and 10, and $s = 2$ with $p = 1, 10, 20$, and 40. The effects of partial pricing can be analyzed with the runs corresponding to $s = 2$, while the effects of scaling can be analyzed when $p = 1$ and $p = 10$. It should be noted that most researchers report the results for $s = 0$ and $p = 1$, the usual default values. (In some cases, the default value for p may not be 1.)

By default, MINOS chooses a triangular initial basis from the matrix $[A \ I]$, as opposed to an initial basis consisting of only the slack variables. Certainly, a different scheme used to choose the initial basis would produce different computational results.

Tables 2.1–2.6 contain the computational results for the 53 problems using the options above. The column for “Phase 1 Its.” is the number of iterations needed to find the first feasible point. The column for “Phase 2 Its.” is the number of iterations needed after the first feasible point has been found. The column for “Total Its.” is the sum of the preceding two columns. The column for “Blocked Its.” is the number of simplex iterations for which movement was blocked, i.e., the number of iterations for which the MINOS routine M5CHZR returned the variable MOVE equal to .FALSE. This happens when a small step is taken and provides a measure of the degree of degeneracy of each problem. The column for “Total Time” is the number of CPU seconds that MINOS used from beginning to end (including reading in the problem data and writing out the solution). The column for “Solution Time” is the number of CPU seconds used by the call to the routine M5SOLV.

3. Analysis of Results

Since linear programs are solved in finite-precision arithmetic, the “optimal” objective value may change depending on the chosen options. Using the optimal value for SCALE OPTION 2 and PARTIAL PRICE 1 for comparison, Table 3.1 indicates the relative error of the optimal objective value reported. Each column represents one scaling and pricing combination, and only nonzero relative errors are given. The first column contains the optimal objective value used for comparison.

To compare the efficiencies of the different options, *performance factors* are used. Let $t_{s,p}$ be the solution time for a specific problem when the MINOS options SCALE OPTION s and PARTIAL PRICE p are used. Then the performance factor

$$f_{s_1,p_1}^{s_2,p_2} = \frac{t_{s_2,p_2}}{t_{s_1,p_1}}$$

measures the efficiency of the option combination (s_2, p_2) relative to the option combination (s_1, p_1) . For example, $f_{0,1}^{2,1} = 0.5$ indicates that when full pricing is used, scaling caused the problem to be solved in half as much time as when scaling was not used.

Performance factors can be computed for each problem to compare any scaling/pricing pairs. Figure 3.2 demonstrates the range of the performance factors $f_{0,1}^{s,p}$ for the different combinations used. The horizontal axis is ordered by problem number, and hence, by the number of nonzeros. Note that a dot representing the value of $f_{0,1}^{0,1}$ (for the default combination of options) is implicitly plotted at the value 1.0 for each problem. All dots below the value 1.0 indicate a combination that performed better than the default combination for MINOS, while dots above the value 1.0 indicate combinations that performed worse than the default combination. It is interesting to note that the factors range from 0.1

to 3.8, indicating that changing the options improved the performance of MINOS by as much as a factor of 10 and worsened the performance by as much as a factor of 3.8. Each vertical line represents one problem, and the ordering of the factors of the six different options varies among the problems, indicating the difficulty of choosing a "best" option. Except for problems 31 and 46 (SCSD6 and SCSD8), the trend seems to indicate that partial pricing is more effective as the number of nonzeros increases. However, for those two problems, the performance deteriorated when scaling was used.

3.1. Effects of Scaling. In order to compare the effects of scaling, the values $f_{0,1}^{2,1}$ and $f_{0,10}^{2,10}$ have been plotted in Figure 3.3. The plot order is determined by sorting on $f_{0,1}^{2,1}$, to make any trend more obvious. This graph indicates that the effects of scaling are usually independent of the partial pricing option chosen. (For problem 49, PILOTJA, the widely separated values are due to the poor performance when scaling was not used.) Also, 71 out of 106 of the tests exhibited a value of $f_{0,p}^{2,p}$ less than 1.0, indicating that scaling improved the performance of MINOS 67% of the time. 50% of the tests exhibited a value less than 0.9, while 84% of the tests exhibited a value less than 1.1. This means that for half the problems, scaling gave a significant improvement, while for most problems, scaling was either better than or comparable to no scaling.

3.2. Effects of Partial Pricing. The effects of partial pricing are analyzed in Figure 3.4 by plotting the values for $f_{2,1}^{2,p}$ for $p = 10, 20$, and 40 . The problems are sorted by $f_{2,1}^{2,10}$ in order to study whether partial pricing improves performance uniformly. Here, we can see that partial pricing does reduce the CPU time for most of the problems. Of course, if the goal were to choose the value of p that minimizes CPU time, the results indicate that determining that value may require extensive experimentation, even for a single problem.

3.3. Iteration Counts. The effects of the different options on iteration counts are analyzed in Figure 3.5 by plotting the values of $\alpha = I/m$, where I is the number of iterations and m is the number of rows. The problems are ordered by the number of rows to reflect any trend in α due to row size. A logarithmic scale was used to demonstrate the range of values of α , which varies from 0.2 to 50. Thus there is a large variance in α , but there is no apparent upward or downward trend. In 35% of the tests, partial pricing reduced the number of iterations as compared to full pricing. Intuitively, one would expect that partial pricing would increase the number of iterations used by the simplex method, since the "best" (in terms of minimal reduced cost) column is not chosen on each iteration.

4. Pictures of Problems

In order to develop a further understanding of the problems in this set, the nonzero structure of each constraint matrix is presented here. Free rows and columns for fixed variables were eliminated. The pictures were produced on an Apple Macintosh Plus, using software developed by the author, and then printed on an Apple LaserWriter, which has a resolution of 300 dots per inch. In order to fit the problems on a page, it was necessary to print the problems at various magnifications. A magnification of 1.0 represents 75 rows/columns per inch, so that a 4x4 box of pixels on the laser printer represents one nonzero. Hence, a picture at a magnification of 0.25 has 300 rows/columns per inch, and every nonzero corresponds to one LaserWriter pixel. The various magnifications and the correspondence between nonzero elements and LaserWriter pixels are summarized in Table 4.1. The ordering of the rows and columns is determined by their ordering in the MPS decks that are created by the program **EMPS**, available from *netlib*.

| Magnification | Rows/Columns Per Inch | Nonzeros | LaserWriter Pixels |
|---------------|--------------------------|----------|-----------------------|
| 10.0 | 7.5 | 1 | 40 |
| 4.0 | 18.75 | 1 | 16 |
| 2.0 | 37.5 | 1 | 8 |
| 1.5 | 50 | 1 | 6 |
| 1.0 | 75 | 1 | 4 |
| 0.5 | 150 | 1 | 2 |
| 0.25 | 300 | 1 | 1 |
| 0.125 | 600 | 2 | 1 |
| 0.0625 | 1200 | 4 | 1 |
| 0.05 | 1500 | 5 | 1 |

Table 4.1 Magnification Correspondences

It is interesting to note that a large number of problems in this test set have staircase structure. Various subsets of the *netlib* problems have been used to compare the simplex method with interior-point algorithms (Karmarkar, 1984). The more favorable results reported for the interior-point approach tend to be associated with strong staircase structure (see, e.g., Adler, Resende and Veiga, 1986; Monma and Morton, 1987). This is fortuitous, since staircase problems have long been viewed as unusually difficult for the simplex method (Fourer, 1982). These problems tend to require many simplex iterations to solve and to have rather dense basis factorizations. It remains to be seen whether most problems of interest in the "real world" display staircase structure.

5. Conclusions

From these results, it is apparent that scaling and partial pricing can improve the performance of the simplex method most of the time. However, when comparing the performance of the simplex method to some other algorithm, care must be taken when choosing these options in order not to degrade the performance of the simplex method.

The pictures of the problems reveal similarities between groups of problems, and indicate that the test set in *netlib* may have some bias which affects these and other computational results.

Acknowledgements

I would like to thank George Dantzig, Philip Gill, Walter Murray, Michael Saunders, and Margaret Wright for their advice and assistance in this research. I would also like to thank the Algorithms Group of the Systems Optimizations Laboratory at Stanford for use of their VAXstation II, and the Department of Operations Research at Stanford for the use of the Apple Macintosh computers, the Apple LaserWriter, Microsoft Excel, and T_EXtures.

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| Prob. # | Problem Name | Rows | Columns | Nonzeroes | Obj. Value (netlib) | Prob. # (netlib) |
|---------|--------------|------|---------|-----------|----------------------|------------------|
| 1 | AFIRO | 28 | 32 | 88 | -4.647531428600E+02 | 1 |
| 2 | ADLITTLE | 57 | 97 | 463 | 2.2549496316000E+05 | 2 |
| 3 | SC205 | 206 | 203 | 552 | -5.2202061211707E+01 | 15f |
| 4 | SCAGR7 | 130 | 140 | 553 | -2.3313892547843E+06 | 17f |
| 5 | SHARE2B | 97 | 79 | 730 | -4.1573224074000E+02 | 3 |
| 6 | RECIPE | 92 | 180 | 752 | -2.6661600000000E+02 | 14f |
| 7 | VTPBASE | 199 | 203 | 914 | 1.2983146246136E+05 | 38f |
| 8 | SHARE1B | 118 | 225 | 1182 | -7.6589318579000E+04 | 4 |
| 9 | BORE3D | 234 | 315 | 1525 | 1.3730803942085E+03 | 1f |
| 10 | SCORPION | 389 | 358 | 1744 | 1.8781248227381E+03 | 21f |
| 11 | CAPRI | 272 | 353 | 1786 | 2.6900129138000E+03 | 9 |
| 12 | SCAGR25 | 472 | 500 | 2029 | -1.4753433060769E+07 | 16f |
| 13 | SCTAP1 | 301 | 480 | 2052 | 1.4122500000000E+03 | 26f |
| 14 | BRANDY | 221 | 249 | 2150 | 1.5185098965000E+03 | 7 |
| 15 | ISRAEL | 175 | 142 | 2358 | -8.9664482186000E+05 | 6 |
| 16 | ETAMACRO | 401 | 688 | 2489 | -7.5571522785000E+02 | 12 |
| 17 | SCFXM1 | 331 | 457 | 2612 | 1.8416759028349E+04 | 18f |
| 18 | GROW7 | 141 | 301 | 2633 | -4.7787811814712E+07 | 8f |
| 19 | BANDM | 306 | 472 | 2659 | -1.5862801845000E+02 | 10 |
| 20 | E226 | 224 | 282 | 2767 | -1.8751929066000E+01 | 8 |
| 21 | STANDATA | 360 | 1075 | 3038 | 1.2576995000000E+03 | 37f |
| 22 | SCSD1 | 78 | 760 | 3148 | 8.6666666743334E+00 | 23f |
| 23 | GFRDPNC | 617 | 1092 | 3467 | 6.9022359995488E+06 | 6f |
| 24 | BEACONFD | 174 | 262 | 3476 | 3.3592485807000E+04 | 5 |
| 25 | STAIR | 357 | 467 | 3857 | -2.5126695119000E+02 | 11 |
| 26 | SCRS8 | 491 | 1169 | 4029 | 9.0429695380079E+02 | 22f |
| 27 | SEBA | 516 | 1028 | 4874 | 1.5711600000000E+04 | 20 |
| 28 | SHELL | 537 | 1775 | 4900 | 1.2088253460000E+09 | 17 |
| 29 | PILOT4 | 411 | 1000 | 5145 | -2.5810162253381E+03 | 11f |
| 30 | SCFXM2 | 661 | 914 | 5229 | 3.6660261564999E+04 | 19f |
| 31 | SCSD6 | 148 | 1350 | 5666 | 5.0500000078262E+01 | 24f |
| 32 | GROW15 | 301 | 645 | 5665 | -1.0687094129358E+08 | 9f |
| 33 | SHIP04S | 403 | 1458 | 5810 | 1.7987147004454E+06 | 30f |
| 34 | FFFFF800 | 525 | 854 | 6235 | 5.5567996085000E+05 | 16 |
| 35 | GANGES | 1310 | 1681 | 7021 | -5.2473581036000E-12 | 18 |
| 36 | SCFXM3 | 991 | 1371 | 7846 | 5.4901254549751E+04 | 20f |
| 37 | SCTAP2 | 1091 | 1880 | 8124 | 1.7248071428571E+03 | 27f |
| 38 | GROW22 | 441 | 946 | 8318 | -1.6083433648256E+08 | 10f |
| 39 | SHIP04L | 403 | 2118 | 8450 | 1.7933245379704E+06 | 29f |
| 40 | PILOTWE | 723 | 2789 | 9218 | -2.7190896937050E+06 | 13f |
| 41 | SIERRA | 1228 | 2036 | 9338 | 1.5394362183632E+07 | 35f |
| 42 | SHIP08S | 779 | 2387 | 9501 | 1.9200982105346E+06 | 32f |
| 43 | SCTAP3 | 1481 | 2480 | 10734 | 1.4240000000000E+03 | 28f |
| 44 | SHIP12S | 1152 | 2763 | 10941 | 1.4892361344061E+06 | 34f |
| 45 | 25FV47 | 822 | 1571 | 11127 | 5.5018458883000E+03 | 14 |
| 46 | SCSD8 | 398 | 2750 | 11334 | 9.0499999992546E+02 | 25f |
| 47 | NESM | 663 | 2923 | 13988 | 1.4076121092176E+07 | 42f |
| 48 | CZPROB | 930 | 3523 | 14173 | 2.1851966989000E+06 | 15 |
| 49 | PILOTJA | 941 | 1988 | 14706 | -6.1100885456374E+03 | 12f |
| 50 | SHIP08L | 779 | 4283 | 17085 | 1.9090552113891E+06 | 31f |
| 51 | SHIP12L | 1152 | 5427 | 21597 | 1.4701879193293E+06 | 33f |
| 52 | 80BAU3B | 2263 | 9799 | 29063 | 9.8722419288205E+05 | 3f |
| 53 | PILOT | 1442 | 3652 | 43220 | -5.5742017351000E+02 | 13 |

Table 1.1 Problem Statistics I

| Problem Name | Rows | | | Columns | | | |
|--------------|--------|-------|---------|---------|------|-------|---------|
| | Normal | Fixed | Bounded | Normal | Free | Fixed | Bounded |
| AFIRO | 19 | 8 | 0 | 32 | 0 | 0 | 0 |
| ADLITTLE | 41 | 15 | 0 | 97 | 0 | 0 | 0 |
| SC205 | 114 | 91 | 0 | 203 | 0 | 0 | 0 |
| SCAGR7 | 45 | 84 | 0 | 140 | 0 | 0 | 0 |
| SHARE2B | 83 | 13 | 0 | 79 | 0 | 0 | 0 |
| RECIPE | 24 | 67 | 0 | 85 | 0 | 26 | 69 |
| VTPBASE | 143 | 55 | 0 | 87 | 1 | 18 | 97 |
| SHARE1B | 28 | 89 | 0 | 225 | 0 | 0 | 0 |
| BORE3D | 19 | 214 | 0 | 302 | 0 | 1 | 12 |
| SCORPION | 108 | 280 | 0 | 358 | 0 | 0 | 0 |
| CAPRI | 129 | 142 | 0 | 192 | 14 | 16 | 131 |
| SCAGR25 | 171 | 300 | 0 | 500 | 0 | 0 | 0 |
| SCTAP1 | 180 | 120 | 0 | 480 | 0 | 0 | 0 |
| BRANDY | 54 | 166 | 0 | 249 | 0 | 0 | 0 |
| ISRAEL | 174 | 0 | 0 | 142 | 0 | 0 | 0 |
| ETAMACRO | 128 | 272 | 0 | 426 | 0 | 82 | 180 |
| SCFXM1 | 143 | 187 | 0 | 457 | 0 | 0 | 0 |
| GROW7 | 0 | 140 | 0 | 21 | 0 | 0 | 280 |
| BANDM | 0 | 305 | 0 | 472 | 0 | 0 | 0 |
| E226 | 190 | 33 | 0 | 282 | 0 | 0 | 0 |
| STANDATA | 199 | 160 | 0 | 955 | 0 | 16 | 104 |
| SCSD1 | 0 | 77 | 0 | 760 | 0 | 0 | 0 |
| GFRDPNC | 68 | 548 | 0 | 834 | 0 | 0 | 258 |
| BEACONFD | 33 | 140 | 0 | 262 | 0 | 0 | 0 |
| STAIR | 147 | 209 | 0 | 373 | 6 | 82 | 6 |
| SCRS8 | 106 | 384 | 0 | 1169 | 0 | 0 | 0 |
| SEBA | 1 | 507 | 7 | 521 | 0 | 0 | 507 |
| SHELL | 2 | 534 | 0 | 1399 | 0 | 250 | 126 |
| PILOT4 | 123 | 287 | 0 | 635 | 88 | 30 | 247 |
| SCFXM2 | 286 | 374 | 0 | 914 | 0 | 0 | 0 |
| SCSD6 | 0 | 147 | 0 | 1350 | 0 | 0 | 0 |
| GROW15 | 0 | 300 | 0 | 45 | 0 | 0 | 600 |
| SHIP04S | 48 | 354 | 0 | 1458 | 0 | 0 | 0 |
| FFFFF800 | 174 | 350 | 0 | 854 | 0 | 0 | 0 |
| GANGES | 25 | 1284 | 0 | 1277 | 0 | 0 | 404 |
| SCFXM3 | 429 | 561 | 0 | 1371 | 0 | 0 | 0 |
| SCTAP2 | 620 | 470 | 0 | 1880 | 0 | 0 | 0 |
| GROW22 | 0 | 440 | 0 | 66 | 0 | 0 | 880 |
| SHIP04L | 48 | 354 | 0 | 2118 | 0 | 0 | 0 |
| PILOTWE | 139 | 583 | 0 | 2335 | 80 | 78 | 296 |
| SIERRA | 699 | 528 | 0 | 0 | 0 | 20 | 2016 |
| SHIP08S | 80 | 698 | 0 | 2387 | 0 | 0 | 0 |
| SCTAP3 | 860 | 620 | 0 | 2480 | 0 | 0 | 0 |
| SHIP12S | 106 | 1045 | 0 | 2763 | 0 | 0 | 0 |
| 25FV47 | 305 | 516 | 0 | 1571 | 0 | 0 | 0 |
| SCSD8 | 0 | 397 | 0 | 2750 | 0 | 0 | 0 |
| NESM | 94 | 480 | 88 | 1009 | 0 | 175 | 1739 |
| CZPROB | 39 | 890 | 0 | 3294 | 0 | 229 | 0 |
| PILOTJA | 279 | 661 | 0 | 1250 | 88 | 311 | 339 |
| SHIP08L | 80 | 698 | 0 | 4283 | 0 | 0 | 0 |
| SHIP12L | 106 | 1045 | 0 | 5427 | 0 | 0 | 0 |
| 80BAU3B | 2262 | 0 | 0 | 6244 | 0 | 498 | 3057 |
| PILOT | 1208 | 233 | 0 | 2320 | 0 | 203 | 1129 |

Table 1.2 Problem Statistics II

| Problem Name | Phase 1 Its. | Phase 2 Its. | Total Its. | Blocked Its. | Total Time | Solution Time |
|--------------|--------------|--------------|------------|--------------|------------|---------------|
| AFIRO | 3 | 6 | 9 | 5 | 2.5 | 0.4 |
| ADLITTLE | 18 | 126 | 144 | 18 | 12.5 | 8.3 |
| SC205 | 0 | 139 | 139 | 14 | 25.8 | 19.0 |
| SCAGR7 | 70 | 21 | 91 | 9 | 13.9 | 8.3 |
| SHARE2B | 57 | 48 | 105 | 22 | 12.8 | 7.6 |
| RECIPE | 7 | 26 | 33 | 3 | 8.6 | 2.2 |
| VTPBASE | 372 | 51 | 423 | 160 | 65.4 | 56.9 |
| SHARE1B | 117 | 169 | 286 | 2 | 42.5 | 34.4 |
| BORE3D | 78 | 33 | 111 | 90 | 30.6 | 19.7 |
| SCORPION | 86 | 52 | 138 | 58 | 44.4 | 31.3 |
| CAPRI | 206 | 114 | 320 | 25 | 72.8 | 59.4 |
| SCAGR25 | 247 | 225 | 472 | 70 | 157.1 | 140.5 |
| SCTAP1 | 168 | 221 | 389 | 106 | 95.1 | 80.3 |
| BRANDY | 178 | 118 | 296 | 36 | 71.7 | 59.6 |
| ISRAEL | 42 | 256 | 298 | 52 | 60.0 | 48.3 |
| ETAMACRO | 305 | 335 | 640 | 210 | 181.1 | 161.6 |
| SCFXM1 | 215 | 178 | 393 | 112 | 108.3 | 91.6 |
| GROW7 | 0 | 167 | 167 | 7 | 53.6 | 39.2 |
| BANDM | 172 | 273 | 445 | 36 | 134.6 | 117.7 |
| E226 | 88 | 493 | 581 | 83 | 134.3 | 119.7 |
| STANDATA | 215 | 158 | 373 | 247 | 130.7 | 107.3 |
| SCSD1 | 81 | 212 | 293 | 127 | 74.2 | 55.4 |
| GFRDPNC | 314 | 368 | 682 | 329 | 284.0 | 254.6 |
| BEACONFD | 33 | 54 | 87 | 6 | 29.3 | 13.5 |
| STAIR | 252 | 274 | 526 | 31 | 254.5 | 233.4 |
| SCRS8 | 113 | 890 | 1003 | 200 | 430.2 | 402.4 |
| SEBA | 146 | 66 | 212 | 31 | 107.9 | 75.3 |
| SHELL | 46 | 212 | 258 | 48 | 143.6 | 105.6 |
| PILOT4 | 954 | 2857 | 3811 | 562 | 2069.3 | 2037.7 |
| SCFXM2 | 504 | 344 | 848 | 235 | 424.5 | 392.1 |
| SCSD6 | 167 | 637 | 804 | 311 | 295.5 | 263.3 |
| GROW15 | 0 | 512 | 512 | 22 | 262.8 | 233.9 |
| SHIP04S | 11 | 137 | 148 | 28 | 91.3 | 57.0 |
| FFFFF800 | 1706 | 321 | 2027 | 615 | 895.0 | 862.4 |
| GANGES | 274 | 404 | 678 | 148 | 484.4 | 432.4 |
| SCFXM3 | 815 | 540 | 1355 | 365 | 1023.7 | 975.7 |
| SCTAP2 | 776 | 727 | 1503 | 613 | 1198.3 | 1144.6 |
| GROW22 | 0 | 901 | 901 | 41 | 630.3 | 588.2 |
| SHIP04L | 10 | 210 | 220 | 37 | 159.1 | 112.4 |
| PILOTWE | 2460 | 13270 | 15730 | 2406 | 15527.7 | 15463.6 |
| SIERRA | 553 | 763 | 1316 | 639 | 1145.8 | 1076.2 |
| SHIP08S | 17 | 229 | 246 | 61 | 231.2 | 174.9 |
| SCTAP3 | 835 | 920 | 1755 | 911 | 1821.6 | 1750.0 |
| SHIP12S | 37 | 411 | 448 | 86 | 478.2 | 410.9 |
| 25FV47 | 2603 | 6469 | 9072 | 720 | 8836.3 | 8779.9 |
| SCSD8 | 433 | 786 | 1219 | 569 | 912.2 | 846.2 |
| NESM | 1642 | 3511 | 5153 | 1 | 4051.3 | 3958.5 |
| CZPROB | 923 | 918 | 1841 | 57 | 1798.4 | 1716.2 |
| PILOTJA | 14948 | 35615 | 50563 | 10613 | 64719.7 | 64643.5 |
| SHIP08L | 12 | 437 | 449 | 68 | 561.4 | 469.2 |
| SHIP12L | 36 | 837 | 873 | 177 | 1344.2 | 1225.7 |
| 80BAU3B | 2047 | 6012 | 8059 | 791 | 19041.2 | 18828.0 |
| PILOT | 22693 | 17554 | 40247 | 1192 | 136832.0 | 136648.6 |

Table 2.1 Results for SCALE OPTION 0 and PARTIAL PRICE 1

| Problem Name | Phase 1 Its. | Phase 2 Its. | Total Its. | Blocked Its. | Total Time | Solution Time |
|--------------|--------------|--------------|------------|--------------|------------|---------------|
| AFIRO | 3 | 6 | 9 | 5 | 2.5 | 0.4 |
| ADLITTLE | 17 | 128 | 145 | 16 | 12.1 | 7.8 |
| SC205 | 0 | 143 | 143 | 14 | 25.7 | 18.7 |
| SCAGR7 | 65 | 25 | 90 | 10 | 13.1 | 7.5 |
| SHARE2B | 62 | 45 | 107 | 22 | 12.8 | 7.5 |
| RECIPE | 7 | 26 | 33 | 3 | 8.3 | 1.9 |
| VTPBASE | 281 | 85 | 366 | 139 | 52.9 | 44.7 |
| SHARE1B | 153 | 130 | 283 | 2 | 37.4 | 29.3 |
| BORE3D | 99 | 30 | 129 | 101 | 32.9 | 21.4 |
| SCORPION | 76 | 50 | 126 | 57 | 42.0 | 28.0 |
| CAPRI | 169 | 105 | 274 | 18 | 59.1 | 45.3 |
| SCAGR25 | 242 | 200 | 442 | 80 | 134.1 | 117.4 |
| SCTAP1 | 326 | 189 | 515 | 123 | 104.4 | 89.3 |
| BRANDY | 214 | 133 | 347 | 27 | 74.7 | 62.6 |
| ISRAEL | 38 | 339 | 377 | 44 | 70.5 | 58.7 |
| ETAMACRO | 310 | 374 | 684 | 231 | 164.2 | 144.6 |
| SCFXM1 | 210 | 220 | 430 | 115 | 100.3 | 83.8 |
| GROW7 | 0 | 166 | 166 | 7 | 55.7 | 41.2 |
| BANDM | 172 | 246 | 418 | 32 | 116.5 | 98.7 |
| E226 | 106 | 579 | 685 | 103 | 136.4 | 122.3 |
| STANDATA | 35 | 58 | 93 | 42 | 40.7 | 17.2 |
| SCSD1 | 66 | 199 | 265 | 204 | 42.5 | 23.7 |
| GFRDPNC | 298 | 563 | 861 | 343 | 300.8 | 271.0 |
| BEACONFD | 34 | 53 | 87 | 6 | 25.7 | 10.3 |
| STAIR | 296 | 292 | 588 | 41 | 257.1 | 235.6 |
| SCRS8 | 284 | 829 | 1113 | 206 | 388.1 | 359.7 |
| SEBA | 287 | 120 | 407 | 64 | 148.5 | 115.6 |
| SHELL | 70 | 270 | 340 | 59 | 127.7 | 88.6 |
| PILOT4 | 910 | 3388 | 4298 | 617 | 2044.9 | 2013.6 |
| SCFXM2 | 542 | 399 | 941 | 203 | 402.3 | 370.3 |
| SCSD6 | 152 | 502 | 654 | 192 | 132.4 | 99.5 |
| GROW15 | 0 | 493 | 493 | 39 | 269.3 | 240.2 |
| SHIP04S | 12 | 165 | 177 | 30 | 74.3 | 40.3 |
| FFFFF800 | 1241 | 383 | 1624 | 435 | 584.8 | 552.3 |
| GANGES | 345 | 402 | 747 | 164 | 469.8 | 416.7 |
| SCFXM3 | 798 | 660 | 1458 | 337 | 930.3 | 882.7 |
| SCTAP2 | 1110 | 924 | 2034 | 799 | 1279.7 | 1224.9 |
| GROW22 | 0 | 748 | 748 | 79 | 547.0 | 504.2 |
| SHIP04L | 14 | 264 | 278 | 50 | 115.9 | 68.9 |
| PILOTWE | 1791 | 10212 | 12003 | 1758 | 9211.8 | 9147.2 |
| SIERRA | 520 | 769 | 1289 | 644 | 816.5 | 747.8 |
| SHIP08S | 19 | 236 | 255 | 62 | 177.1 | 121.3 |
| SCTAP3 | 1169 | 899 | 2068 | 988 | 1699.3 | 1626.6 |
| SHIP12S | 41 | 392 | 433 | 80 | 335.8 | 268.3 |
| 25FV47 | 2491 | 7310 | 9801 | 729 | 7911.9 | 7855.1 |
| SCSD8 | 554 | 960 | 1514 | 568 | 671.0 | 605.2 |
| NESM | 1708 | 2456 | 4164 | 5 | 2126.4 | 2033.9 |
| CZPROB | 754 | 921 | 1675 | 62 | 1079.3 | 996.8 |
| PILOTJA | 12814 | 28395 | 41209 | 7345 | 45598.1 | 45521.0 |
| SHIP08L | 13 | 426 | 439 | 60 | 351.1 | 258.8 |
| SHIP12L | 39 | 856 | 895 | 180 | 778.5 | 659.6 |
| 80BAU3B | 1698 | 5515 | 7213 | 774 | 9327.7 | 9114.7 |
| PILOT | 14851 | 16384 | 31235 | 1162 | 93568.3 | 93384.7 |

Table 2.2 Results for SCALE OPTION 0 and PARTIAL PRICE 10

| Problem Name | Phase 1 Its. | Phase 2 Its. | Total Its. | Blocked Its. | Total Time | Solution Time |
|--------------|--------------|--------------|------------|--------------|------------|---------------|
| AFIRO | 2 | 4 | 6 | 3 | 2.6 | 0.3 |
| ADLITTLE | 19 | 95 | 114 | 13 | 12.0 | 7.2 |
| SC205 | 0 | 110 | 110 | 14 | 22.3 | 14.6 |
| SCAGR7 | 60 | 28 | 88 | 10 | 14.3 | 8.1 |
| SHARE2B | 54 | 61 | 115 | 20 | 14.4 | 9.0 |
| RECIPE | 7 | 26 | 33 | 3 | 9.4 | 2.2 |
| VTPBASE | 55 | 34 | 89 | 38 | 20.4 | 10.9 |
| SHARE1B | 156 | 118 | 274 | 10 | 40.6 | 31.3 |
| BORE3D | 101 | 47 | 148 | 85 | 39.3 | 26.5 |
| SCORPION | 70 | 34 | 104 | 44 | 38.3 | 23.0 |
| CAPRI | 145 | 100 | 245 | 25 | 57.3 | 42.1 |
| SCAGR25 | 151 | 156 | 307 | 48 | 105.4 | 86.7 |
| SCTAP1 | 107 | 109 | 216 | 60 | 61.4 | 44.3 |
| BRANDY | 296 | 181 | 477 | 28 | 111.5 | 97.0 |
| ISRAEL | 34 | 262 | 296 | 17 | 63.3 | 49.9 |
| ETAMACRO | 350 | 268 | 618 | 122 | 190.8 | 167.6 |
| SCFXM1 | 183 | 132 | 315 | 41 | 92.9 | 73.5 |
| GROW7 | 0 | 160 | 160 | 6 | 53.2 | 36.0 |
| BANDM | 233 | 221 | 454 | 28 | 139.7 | 120.5 |
| E226 | 154 | 318 | 472 | 91 | 112.9 | 96.6 |
| STANDATA | 70 | 59 | 129 | 74 | 63.6 | 36.0 |
| SCSD1 | 76 | 547 | 623 | 394 | 145.3 | 124.8 |
| GFRDPNC | 266 | 393 | 659 | 271 | 288.2 | 251.5 |
| BEACONFD | 40 | 58 | 98 | 17 | 33.1 | 15.4 |
| STAIR | 300 | 118 | 418 | 52 | 253.4 | 228.2 |
| SCRS8 | 109 | 559 | 668 | 180 | 302.3 | 269.5 |
| SEBA | 217 | 147 | 364 | 58 | 158.6 | 122.1 |
| SHELL | 46 | 212 | 258 | 48 | 150.2 | 107.3 |
| PILOT4 | 523 | 1010 | 1533 | 186 | 841.0 | 804.6 |
| SCFXM2 | 534 | 340 | 874 | 118 | 441.2 | 402.2 |
| SCSD6 | 168 | 1393 | 1561 | 606 | 553.4 | 516.3 |
| GROW15 | 0 | 464 | 464 | 12 | 235.6 | 201.3 |
| SHIP04S | 10 | 134 | 144 | 25 | 95.7 | 56.5 |
| FFFFF800 | 800 | 139 | 939 | 346 | 435.1 | 395.2 |
| GANGES | 495 | 204 | 699 | 204 | 529.5 | 470.1 |
| SCFXM3 | 761 | 462 | 1223 | 183 | 915.2 | 855.6 |
| SCTAP2 | 256 | 497 | 753 | 419 | 602.5 | 541.3 |
| GROW22 | 0 | 756 | 756 | 23 | 507.7 | 456.6 |
| SHIP04L | 10 | 221 | 231 | 36 | 171.7 | 117.8 |
| PILOTWE | 350 | 6346 | 6696 | 1090 | 6457.3 | 6379.7 |
| SIERRA | 614 | 737 | 1351 | 552 | 1148.9 | 1060.0 |
| SHIP08S | 17 | 223 | 240 | 60 | 232.2 | 167.8 |
| SCTAP3 | 296 | 648 | 944 | 564 | 988.3 | 906.2 |
| SHIP12S | 36 | 363 | 399 | 68 | 436.0 | 359.1 |
| 25FV47 | 2309 | 6133 | 8442 | 714 | 7684.4 | 7618.6 |
| SCSD8 | 580 | 3755 | 4335 | 2722 | 3288.1 | 3214.0 |
| NESM | 1200 | 1687 | 2887 | 0 | 2377.5 | 2271.6 |
| CZPROB | 720 | 805 | 1525 | 94 | 1519.5 | 1421.0 |
| PILOTJA | 1939 | 5175 | 7114 | 613 | 7751.9 | 7664.9 |
| SHIP08L | 12 | 437 | 449 | 73 | 576.8 | 470.9 |
| SHIP12L | 36 | 833 | 869 | 171 | 1384.4 | 1248.7 |
| 80BAU3B | 1525 | 15941 | 17466 | 1177 | 37894.1 | 37642.0 |
| PILOT | 11460 | 6705 | 18165 | 2044 | 84860.0 | 84640.5 |

Table 2.3 Results with SCALE OPTION 2 and PARTIAL PRICE 1

| Problem Name | Phase 1 Its. | Phase 2 Its. | Total Its. | Blocked Its. | Total Time | Solution Time |
|--------------|--------------|--------------|------------|--------------|------------|---------------|
| AFIRO | 2 | 4 | 6 | 3 | 2.7 | 0.3 |
| ADLITTLE | 25 | 67 | 92 | 12 | 9.8 | 5.0 |
| SC205 | 0 | 114 | 114 | 12 | 22.0 | 14.5 |
| SCAGR7 | 63 | 23 | 86 | 10 | 13.3 | 7.1 |
| SHARE2B | 63 | 53 | 116 | 20 | 13.7 | 7.7 |
| RECIPE | 7 | 26 | 33 | 3 | 9.3 | 2.0 |
| VTPBASE | 73 | 30 | 103 | 43 | 21.2 | 11.4 |
| SHARE1B | 165 | 104 | 269 | 8 | 34.4 | 25.5 |
| BORE3D | 138 | 54 | 192 | 98 | 41.5 | 28.5 |
| SCORPION | 70 | 40 | 110 | 42 | 39.5 | 24.4 |
| CAPRI | 163 | 86 | 249 | 23 | 52.5 | 37.3 |
| SCAGR25 | 175 | 186 | 361 | 68 | 111.6 | 93.1 |
| SCTAP1 | 159 | 143 | 302 | 62 | 64.9 | 48.7 |
| BRANDY | 270 | 210 | 480 | 40 | 101.3 | 86.9 |
| ISRAEL | 16 | 226 | 242 | 17 | 48.2 | 34.7 |
| ETAMACRO | 286 | 200 | 486 | 112 | 128.2 | 105.0 |
| SCFXM1 | 256 | 151 | 407 | 62 | 98.4 | 78.7 |
| GROW7 | 0 | 175 | 175 | 19 | 59.1 | 41.9 |
| BANDM | 212 | 252 | 464 | 25 | 126.1 | 107.1 |
| E226 | 122 | 415 | 537 | 91 | 106.5 | 90.3 |
| STANDATA | 37 | 65 | 102 | 42 | 46.5 | 19.8 |
| SCSD1 | 91 | 348 | 439 | 227 | 62.4 | 41.4 |
| GFRDPNC | 243 | 446 | 689 | 284 | 244.7 | 208.6 |
| BEACONFD | 37 | 79 | 116 | 21 | 32.6 | 14.7 |
| STAIR | 305 | 133 | 438 | 42 | 240.2 | 215.0 |
| SCRS8 | 115 | 494 | 609 | 148 | 215.4 | 183.5 |
| SEBA | 268 | 172 | 440 | 59 | 148.4 | 111.9 |
| SHELL | 70 | 235 | 305 | 55 | 126.1 | 82.9 |
| PILOT4 | 434 | 1216 | 1650 | 169 | 798.0 | 762.2 |
| SCFXM2 | 530 | 371 | 901 | 127 | 376.8 | 337.4 |
| SCSD6 | 201 | 1149 | 1350 | 720 | 265.0 | 227.8 |
| GROW15 | 0 | 430 | 430 | 58 | 232.0 | 196.7 |
| SHIP04S | 13 | 138 | 151 | 27 | 77.2 | 38.2 |
| FFFFF800 | 956 | 234 | 1190 | 304 | 440.3 | 401.0 |
| GANGES | 503 | 247 | 750 | 202 | 491.7 | 432.4 |
| SCFXM3 | 837 | 483 | 1320 | 218 | 804.0 | 744.7 |
| SCTAP2 | 282 | 502 | 784 | 399 | 475.9 | 414.5 |
| GROW22 | 0 | 657 | 657 | 65 | 465.7 | 414.7 |
| SHIP04L | 13 | 264 | 277 | 47 | 124.0 | 70.8 |
| PILOTWE | 480 | 6217 | 6697 | 899 | 4931.9 | 4853.7 |
| SIERRA | 683 | 654 | 1337 | 594 | 854.7 | 765.7 |
| SHIP08S | 17 | 252 | 269 | 62 | 194.0 | 129.8 |
| SCTAP3 | 401 | 728 | 1129 | 688 | 909.6 | 827.0 |
| SHIP12S | 39 | 393 | 432 | 77 | 362.2 | 284.6 |
| 25FV47 | 2583 | 6633 | 9216 | 734 | 6697.7 | 6631.1 |
| SCSD8 | 685 | 3124 | 3809 | 2179 | 1745.3 | 1670.8 |
| NESM | 1275 | 1951 | 3226 | 1 | 1611.6 | 1506.3 |
| CZPROB | 707 | 900 | 1607 | 98 | 1020.5 | 921.6 |
| PILOTJA | 1882 | 5099 | 6981 | 607 | 6440.1 | 6353.4 |
| SHIP08L | 12 | 466 | 478 | 70 | 363.1 | 257.2 |
| SHIP12L | 38 | 814 | 852 | 165 | 779.0 | 642.3 |
| 80BAU3B | 1580 | 9447 | 11027 | 1070 | 13917.3 | 13664.8 |
| PILOT | 12254 | 6681 | 18935 | 4220 | 80158.6 | 79939.9 |

Table 2.4 Results with SCALE OPTION 2 and PARTIAL PRICE 10

| Problem Name | Phase 1 Its. | Phase 2 Its. | Total Its. | Blocked Its. | Total Time | Solution Time |
|--------------|--------------|--------------|------------|--------------|------------|---------------|
| AFIRO | 2 | 4 | 6 | 3 | 2.6 | 0.3 |
| ADLITTLE | 33 | 94 | 127 | 18 | 11.6 | 6.9 |
| SC205 | 0 | 120 | 120 | 23 | 22.1 | 15.0 |
| SCAGR7 | 56 | 28 | 84 | 8 | 13.4 | 7.1 |
| SHARE2B | 70 | 54 | 124 | 25 | 14.5 | 8.5 |
| RECIPE | 7 | 28 | 35 | 3 | 9.2 | 2.0 |
| VTPBASE | 40 | 22 | 62 | 22 | 16.6 | 6.8 |
| SHARE1B | 120 | 137 | 257 | 11 | 33.5 | 24.5 |
| BORE3D | 115 | 41 | 156 | 77 | 35.9 | 22.9 |
| SCORPION | 66 | 39 | 105 | 41 | 37.2 | 22.0 |
| CAPRI | 162 | 95 | 257 | 26 | 53.1 | 37.9 |
| SCAGR25 | 162 | 209 | 371 | 64 | 111.8 | 93.2 |
| SCTAP1 | 169 | 120 | 289 | 65 | 61.9 | 45.7 |
| BRANDY | 277 | 213 | 490 | 34 | 100.8 | 86.5 |
| ISRAEL | 19 | 276 | 295 | 15 | 55.5 | 42.1 |
| ETAMACRO | 260 | 272 | 532 | 125 | 135.4 | 112.3 |
| SCFXM1 | 246 | 172 | 418 | 63 | 97.7 | 78.2 |
| GROW7 | 0 | 175 | 175 | 14 | 56.3 | 39.3 |
| BANDM | 202 | 268 | 470 | 34 | 126.1 | 107.1 |
| E226 | 165 | 317 | 482 | 99 | 93.3 | 77.1 |
| STANDATA | 72 | 94 | 166 | 82 | 56.4 | 29.5 |
| SCSD1 | 74 | 284 | 358 | 202 | 51.5 | 31.0 |
| GFRDPNC | 265 | 448 | 713 | 277 | 245.9 | 209.8 |
| BEACONFD | 66 | 67 | 133 | 24 | 34.0 | 16.2 |
| STAIR | 279 | 103 | 382 | 40 | 216.7 | 191.5 |
| SCRS8 | 95 | 482 | 577 | 201 | 196.0 | 163.3 |
| SEBA | 306 | 183 | 489 | 70 | 160.7 | 124.9 |
| SHELL | 77 | 240 | 317 | 59 | 123.5 | 80.4 |
| PILOT4 | 510 | 1120 | 1630 | 121 | 789.0 | 752.7 |
| SCFXM2 | 590 | 305 | 895 | 120 | 371.5 | 332.0 |
| SCSD6 | 186 | 1231 | 1417 | 574 | 245.0 | 207.7 |
| GROW15 | 0 | 429 | 429 | 40 | 231.5 | 196.3 |
| SHIP04S | 15 | 148 | 163 | 32 | 76.1 | 37.0 |
| FFFFF800 | 906 | 235 | 1141 | 281 | 404.4 | 364.6 |
| GANGES | 552 | 218 | 770 | 196 | 489.2 | 429.4 |
| SCFXM3 | 857 | 523 | 1380 | 245 | 831.7 | 772.2 |
| SCTAP2 | 312 | 529 | 841 | 396 | 493.2 | 431.5 |
| GROW22 | 0 | 737 | 737 | 48 | 497.7 | 446.4 |
| SHIP04L | 15 | 254 | 269 | 43 | 115.3 | 61.9 |
| PILOTWE | 587 | 4058 | 4645 | 574 | 3369.7 | 3291.2 |
| SIERRA | 677 | 675 | 1352 | 609 | 823.2 | 734.3 |
| SHIP08S | 17 | 251 | 268 | 70 | 178.0 | 113.6 |
| SCTAP3 | 432 | 738 | 1170 | 655 | 896.2 | 814.8 |
| SHIP12S | 42 | 392 | 434 | 77 | 340.3 | 263.1 |
| 25FV47 | 2634 | 5695 | 8329 | 757 | 5830.0 | 5763.8 |
| SCSD8 | 970 | 2959 | 3929 | 1888 | 1616.5 | 1541.8 |
| NESM | 1261 | 1529 | 2790 | 4 | 1286.4 | 1180.9 |
| CZPROB | 619 | 913 | 1532 | 84 | 932.7 | 833.3 |
| PILOTJA | 1907 | 4630 | 6537 | 894 | 5961.1 | 5874.1 |
| SHIP08L | 12 | 507 | 519 | 88 | 352.8 | 246.7 |
| SHIP12L | 44 | 811 | 855 | 155 | 715.2 | 579.2 |
| 80BAU3B | 1834 | 9289 | 11123 | 1223 | 13325.9 | 13073.7 |
| PILOT | 13352 | 7870 | 21222 | 4667 | 86119.7 | 85900.0 |

Table 2.5 Results with SCALE OPTION 2 and PARTIAL PRICE 20

| Problem Name | Phase 1 Its. | Phase 2 Its. | Total Its. | Blocked Its. | Total Time | Solution Time |
|--------------|--------------|--------------|------------|--------------|------------|---------------|
| AFIRO | 2 | 4 | 6 | 3 | 2.6 | 0.3 |
| ADLITTLE | 35 | 89 | 124 | 19 | 11.4 | 6.7 |
| SC205 | 0 | 130 | 130 | 22 | 24.5 | 16.9 |
| SCAGR7 | 51 | 29 | 80 | 11 | 12.2 | 6.1 |
| SHARE2B | 62 | 36 | 98 | 15 | 12.3 | 6.6 |
| RECIPE | 7 | 28 | 35 | 3 | 9.1 | 1.9 |
| VITBASE | 62 | 31 | 93 | 36 | 19.4 | 9.6 |
| SHARE1B | 148 | 167 | 315 | 7 | 38.8 | 30.1 |
| BORE3D | 126 | 52 | 178 | 90 | 39.4 | 26.4 |
| SCORPION | 75 | 44 | 119 | 41 | 40.9 | 25.6 |
| CAPRI | 152 | 114 | 266 | 29 | 54.0 | 38.8 |
| SCAGR25 | 171 | 200 | 371 | 70 | 110.5 | 92.0 |
| SCTAP1 | 182 | 140 | 322 | 64 | 66.3 | 49.5 |
| BRANDY | 326 | 191 | 517 | 33 | 104.4 | 90.5 |
| ISRAEL | 14 | 238 | 252 | 15 | 49.5 | 36.2 |
| ETAMACRO | 258 | 245 | 503 | 142 | 125.8 | 102.7 |
| SCFXM1 | 255 | 153 | 408 | 70 | 98.1 | 78.3 |
| GROW7 | 0 | 176 | 176 | 17 | 56.5 | 40.0 |
| BANDM | 191 | 196 | 387 | 23 | 105.8 | 87.3 |
| E226 | 175 | 443 | 618 | 138 | 115.3 | 99.1 |
| STANDATA | 95 | 91 | 186 | 93 | 58.4 | 31.0 |
| SCSD1 | 90 | 377 | 467 | 306 | 59.2 | 38.1 |
| GFRDPNC | 259 | 519 | 778 | 300 | 260.0 | 223.8 |
| BEACONFD | 80 | 61 | 141 | 22 | 34.9 | 17.1 |
| STAIR | 276 | 122 | 398 | 31 | 227.5 | 202.8 |
| SCRS8 | 141 | 458 | 599 | 185 | 205.1 | 172.9 |
| SEBA | 219 | 157 | 376 | 44 | 125.9 | 89.3 |
| SHELL | 105 | 241 | 346 | 63 | 129.8 | 86.8 |
| PILOT4 | 573 | 1114 | 1687 | 138 | 811.3 | 775.5 |
| SCFXM2 | 504 | 311 | 815 | 129 | 336.8 | 297.3 |
| SCSD6 | 237 | 815 | 1052 | 318 | 184.3 | 147.8 |
| GROW15 | 0 | 472 | 472 | 42 | 249.9 | 214.6 |
| SHIP04S | 19 | 156 | 175 | 33 | 77.1 | 38.2 |
| FFFFF800 | 809 | 299 | 1108 | 354 | 388.3 | 348.5 |
| GANGES | 542 | 229 | 771 | 198 | 478.7 | 418.7 |
| SCFXM3 | 816 | 497 | 1313 | 217 | 784.4 | 724.6 |
| SCTAP2 | 364 | 608 | 972 | 453 | 561.0 | 499.3 |
| GROW22 | 0 | 641 | 641 | 59 | 437.7 | 386.8 |
| SHIP04L | 33 | 272 | 305 | 45 | 118.1 | 64.3 |
| PILOTWE | 492 | 4377 | 4869 | 490 | 3498.8 | 3419.8 |
| SIERRA | 610 | 682 | 1292 | 542 | 772.9 | 683.7 |
| SHIP08S | 17 | 253 | 270 | 63 | 173.2 | 109.5 |
| SCTAP3 | 430 | 742 | 1172 | 643 | 897.4 | 815.6 |
| SHIP12S | 43 | 373 | 416 | 71 | 342.0 | 264.9 |
| 25FV47 | 2162 | 5137 | 7299 | 684 | 5044.6 | 4978.5 |
| SCSD8 | 929 | 2767 | 3696 | 1825 | 1419.2 | 1345.1 |
| NESM | 1280 | 1153 | 2433 | 3 | 1055.8 | 950.0 |
| CZPROB | 744 | 912 | 1656 | 95 | 976.5 | 877.7 |
| PILOTJA | 2052 | 4948 | 7000 | 537 | 6276.7 | 6190.1 |
| SHIP08L | 12 | 522 | 534 | 87 | 342.6 | 236.7 |
| SHIP12L | 46 | 844 | 890 | 175 | 723.7 | 588.5 |
| 80BAU3B | 1503 | 9153 | 10656 | 1006 | 12000.6 | 11747.8 |
| PILOT | 11660 | 6701 | 18361 | 4234 | 71991.3 | 71772.9 |

Table 2.6 Results with SCALE OPTION 2 and PARTIAL PRICE 40

| Problem Name | Scaled PP 1 Objective Value | Scaled PP 10 | Scaled PP 20 | Scaled PP 40 | Unscaled PP 1 | Unscaled PP 10 |
|--------------|-----------------------------|--------------|--------------|--------------|---------------|----------------|
| AFIRO | -4.6475314285714E+02 | | | | | |
| ADLITTLE | 2.2549496316238E+05 | | | | | |
| SC205 | -5.2202061211707E+01 | | | | | |
| SCAGR7 | -2.3313898243310E+06 | 3.1E-08 | 3.1E-08 | 3.1E-08 | 3.1E-08 | |
| SHARE2B | -4.1573224074142E+02 | | | | | |
| RECIPE | -2.6661600000000E+02 | | | | | |
| VTPBASE | 1.2983146246136E+05 | | | | | |
| SHARE1B | -7.6589318579186E+04 | | | | | |
| BORE3D | 1.3730803942085E+03 | | | | | |
| SCORPION | 1.8781248227381E+03 | | | | | |
| CAPRI | 2.6900129137682E+03 | | | | | |
| SCAGR25 | -1.4753433060769E+07 | | | | | |
| SCTAP1 | 1.4122500000000E+03 | | | | | |
| BRANDY | 1.5185098964881E+03 | | | | | |
| ISRAEL | -8.9664482186305E+05 | | | | | |
| ETAMACRO | -7.5571521856065E+02 | 5.9E-10 | 1.1E-09 | 6.4E-10 | 7.1E-09 | 7.5E-09 |
| SCFXM1 | 1.8416759028349E+04 | | | | | |
| GROW7 | -4.7787811814712E+07 | | | | | |
| BANDM | -1.5862801845012E+02 | | | | | |
| E226 | -1.8751929066371E+01 | | | | | |
| STANDATA | 1.2576995000000E+03 | | | | | |
| SCSD1 | 8.6666666743334E+00 | | | | | |
| GFRDPNC | 6.9022359995488E+06 | | | | | |
| BEACONFD | 3.3592485807200E+04 | | | | | |
| STAIR | -2.5126695119296E+02 | | | | | |
| SCRS8 | 9.0429998618888E+02 | | 3.4E-06 | 3.4E-06 | 3.4E-06 | 3.4E-06 |
| SEBA | 1.5711600000000E+04 | | | | | |
| SHELL | 1.2088253460000E+09 | | | | | |
| PILOT4 | -2.5811397876813E+03 | 3.5E-13 | 1.7E-09 | 2.9E-10 | 4.8E-05 | 3.9E-05 |
| SCFXM2 | 3.6660261564999E+04 | | | | | |
| SCSD6 | 5.0500000078262E+01 | | | | | |
| GROW15 | -1.0687094129358E+08 | | | | | |
| SHIP04S | 1.7987147004454E+06 | | | | | |
| FFFFF800 | 5.5567959102690E+05 | 4.0E-08 | 3.6E-08 | 3.6E-08 | 6.7E-07 | 4.6E-05 |
| GANGES | -1.0958576909180E+05 | 3.4E-08 | 6.2E-08 | 6.2E-08 | 3.0E-07 | 3.0E-07 |
| SCFXM3 | 5.4901254549751E+04 | | | | | |
| SCTAP2 | 1.7248071428571E+03 | | | | | |
| GROW22 | -1.6083433648256E+08 | | | | | |
| SHIP04L | 1.7933245379704E+06 | | | | | |
| PILOTWE | -2.7200979898204E+06 | 1.9E-06 | 2.4E-07 | 2.1E-07 | 3.7E-04 | 5.2E-04 |
| SIERRA | 1.5394362183632E+07 | | | | 3.9E-04 | 2.0E-06 |
| SHIP08S | 1.9200982105346E+06 | | | | | |
| SCTAP3 | 1.4240000000000E+03 | | | | | |
| SHIP12S | 1.4892361344061E+06 | | | | | |
| 25FV47 | 5.5018458882867E+03 | | | | | |
| SCSD8 | 9.0499999992546E+02 | | | | | |
| NESM | 1.4076086885553E+07 | 6.1E-07 | 9.4E-08 | 4.9E-07 | 1.1E-06 | 1.4E-06 |
| CZPROB | 2.1851966988566E+06 | | | | | |
| PILOTJA | -6.1130625520046E+03 | | | 3.0E-10 | 4.7E-04 | 4.6E-04 |
| SHIP08L | 1.9090552113891E+06 | | | | | |
| SHIP12L | 1.4701879193293E+06 | | | | | |
| 80BAU3B | 9.8722925484290E+05 | 6.1E-07 | 2.3E-06 | 1.7E-07 | 5.1E-06 | 5.1E-06 |
| PILOT | -5.5740380063326E+02 | 4.7E-06 | | | 6.5E-05 | 7.5E-05 |

Table 3.1 Relative Error in Objective Compared to SCALE OPTION 2 and PARTIAL PRICE 1

Figure 3.2
Solution Time Factors Compared to No Scaling, Full Pricing

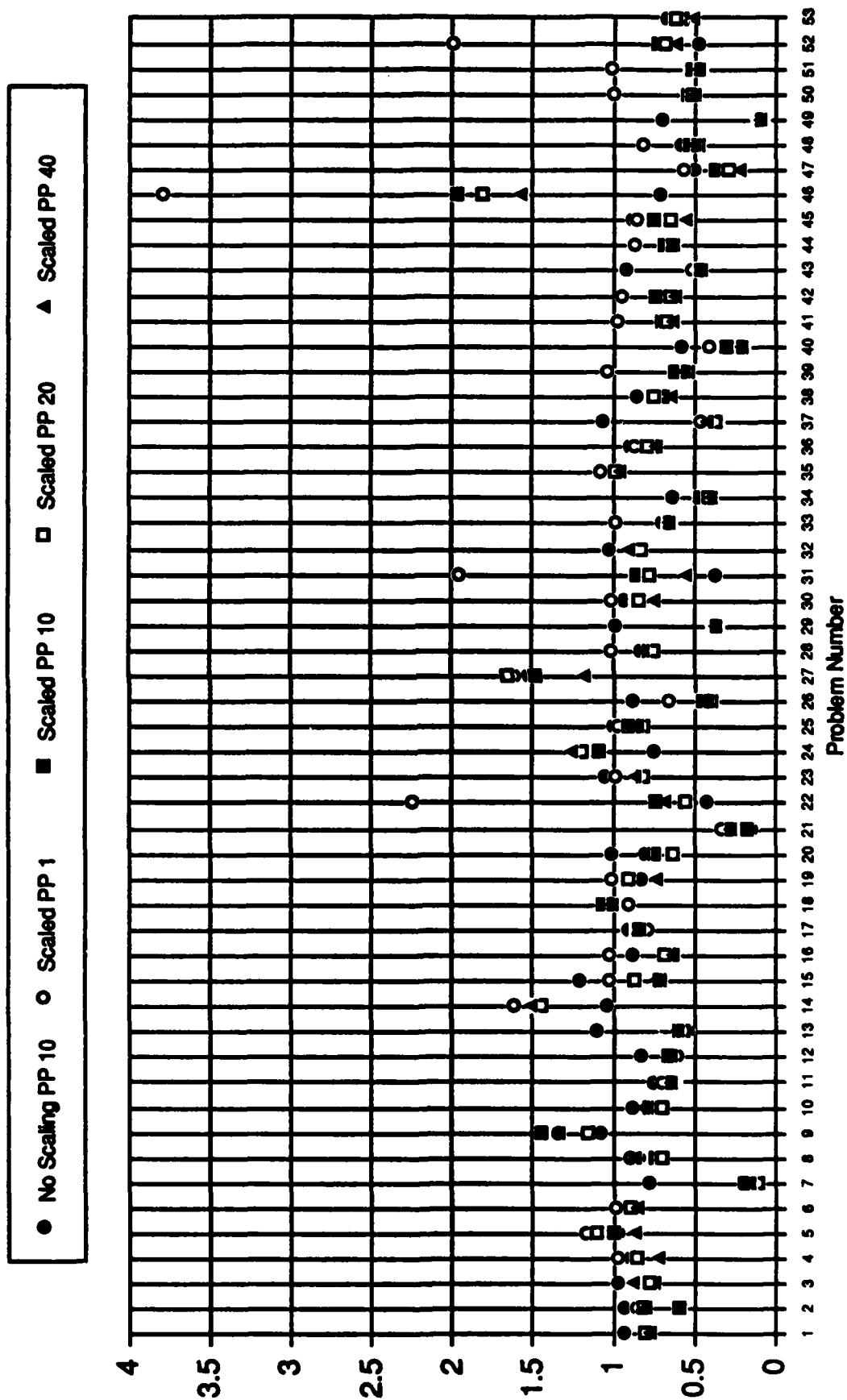


Figure 3.3
Solution Time Factors Compared to No Scaling Equivalents
Sorted by Factor for Full Pricing

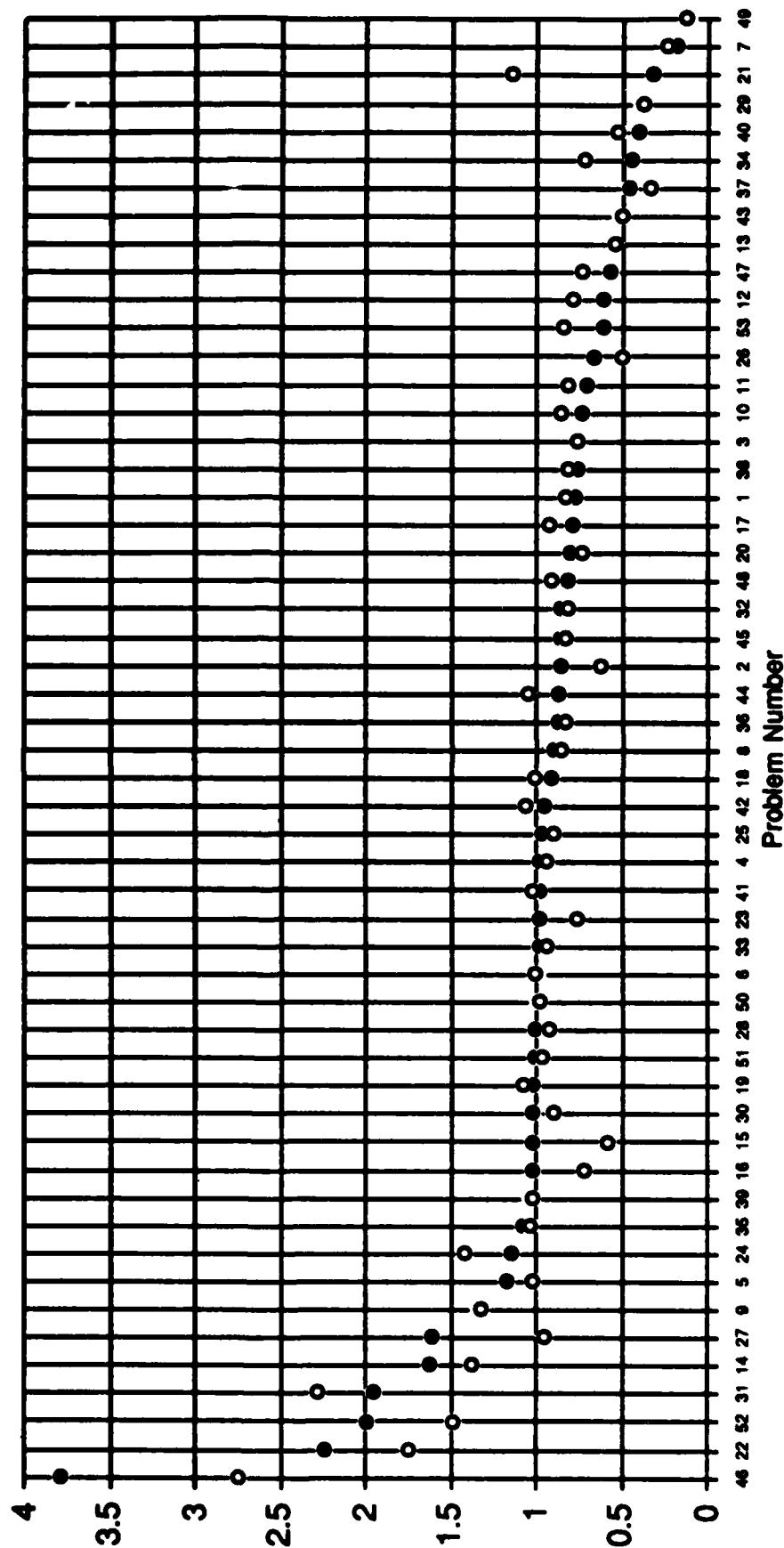
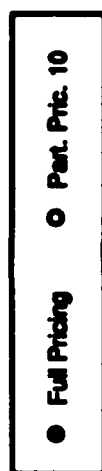


Figure 3.4
Solution Time Factors Compared to Scaling, Full Pricing
Ordered by Factor for Partial Price 10

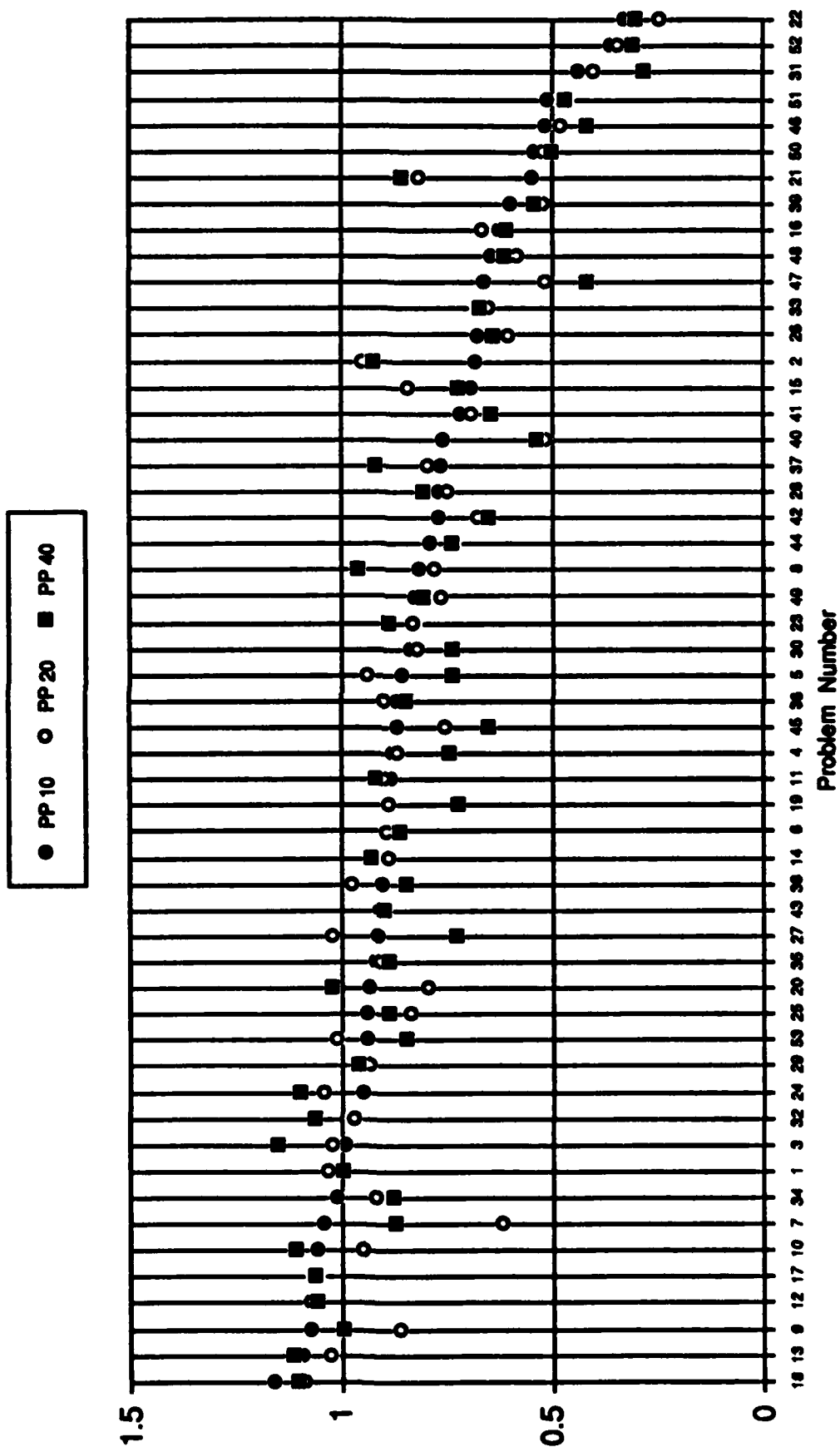
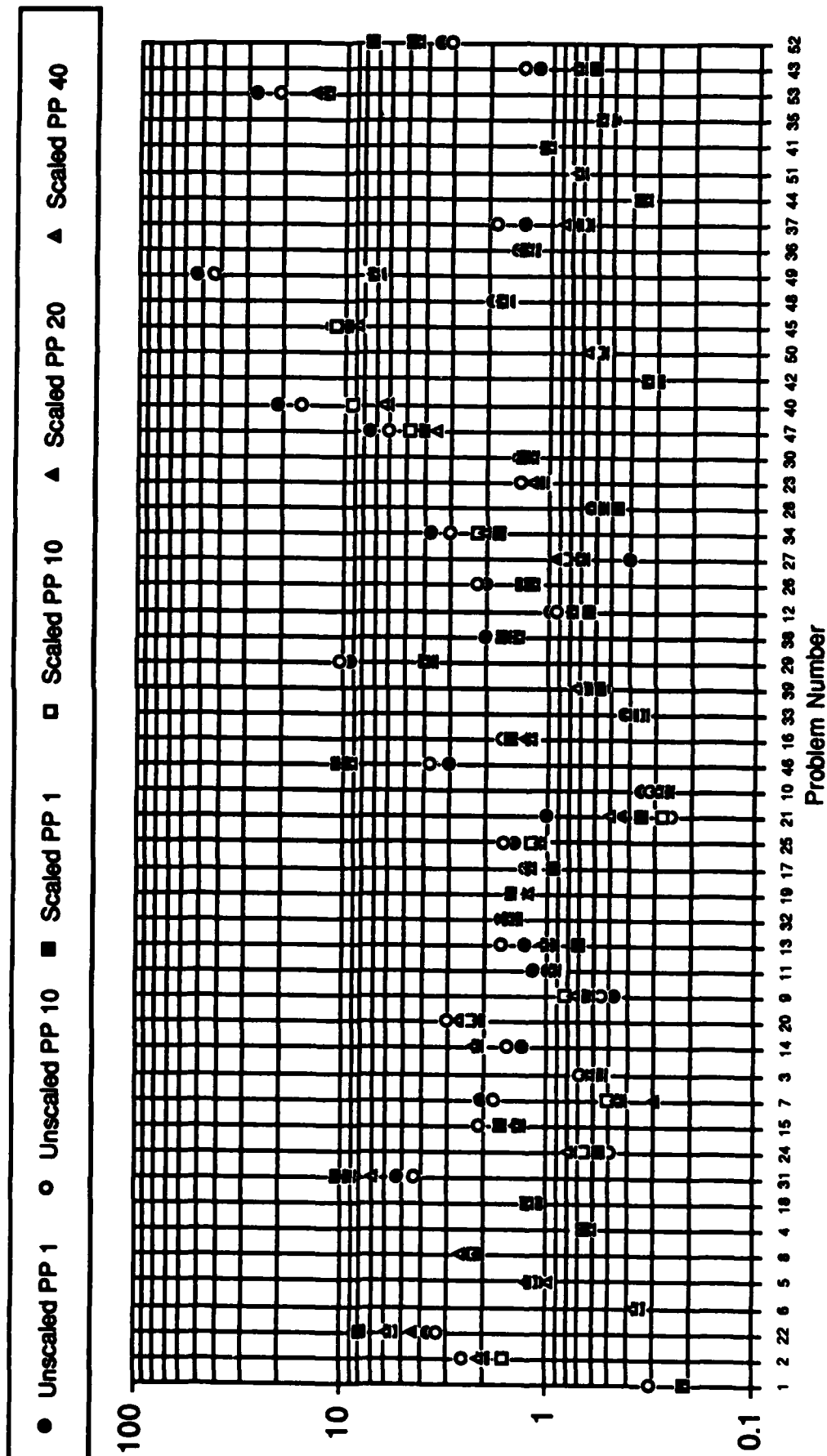
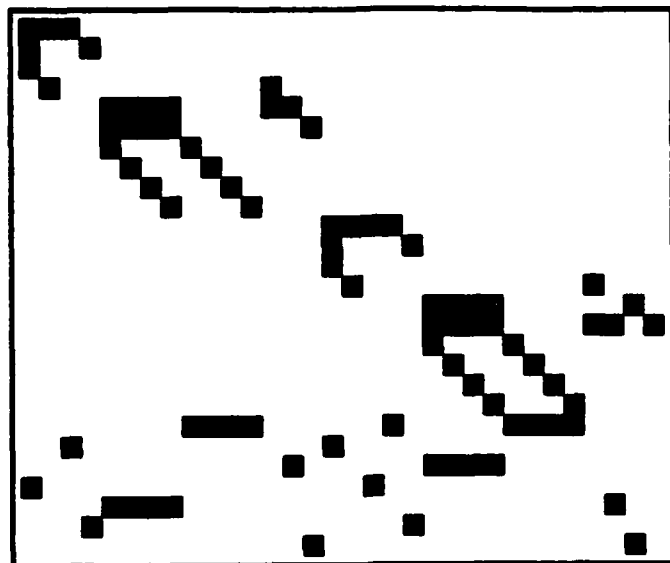
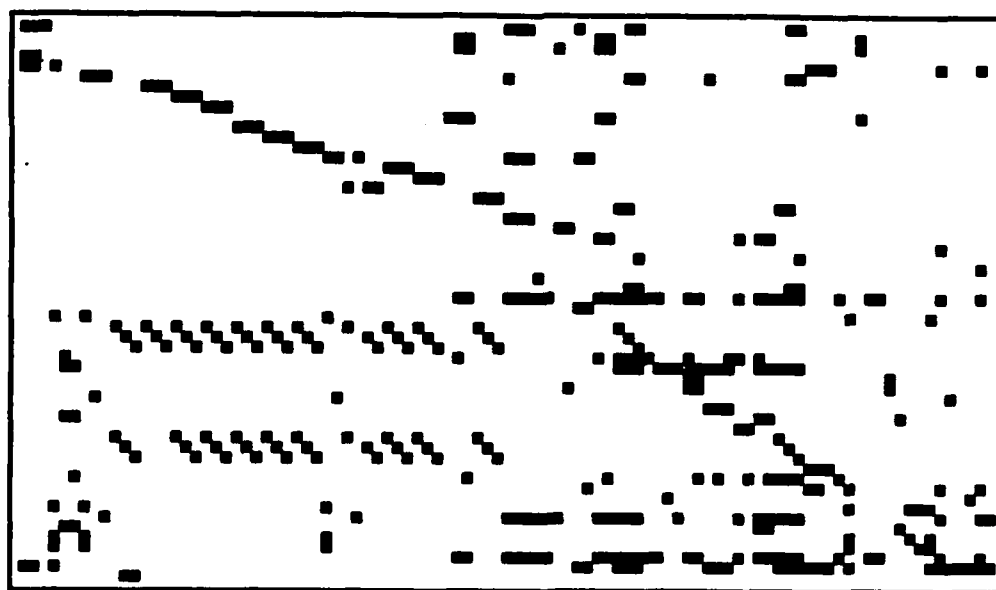


Figure 3.5
Ratio of Iteration Count to Number of Rows
Ordered by Number of Rows

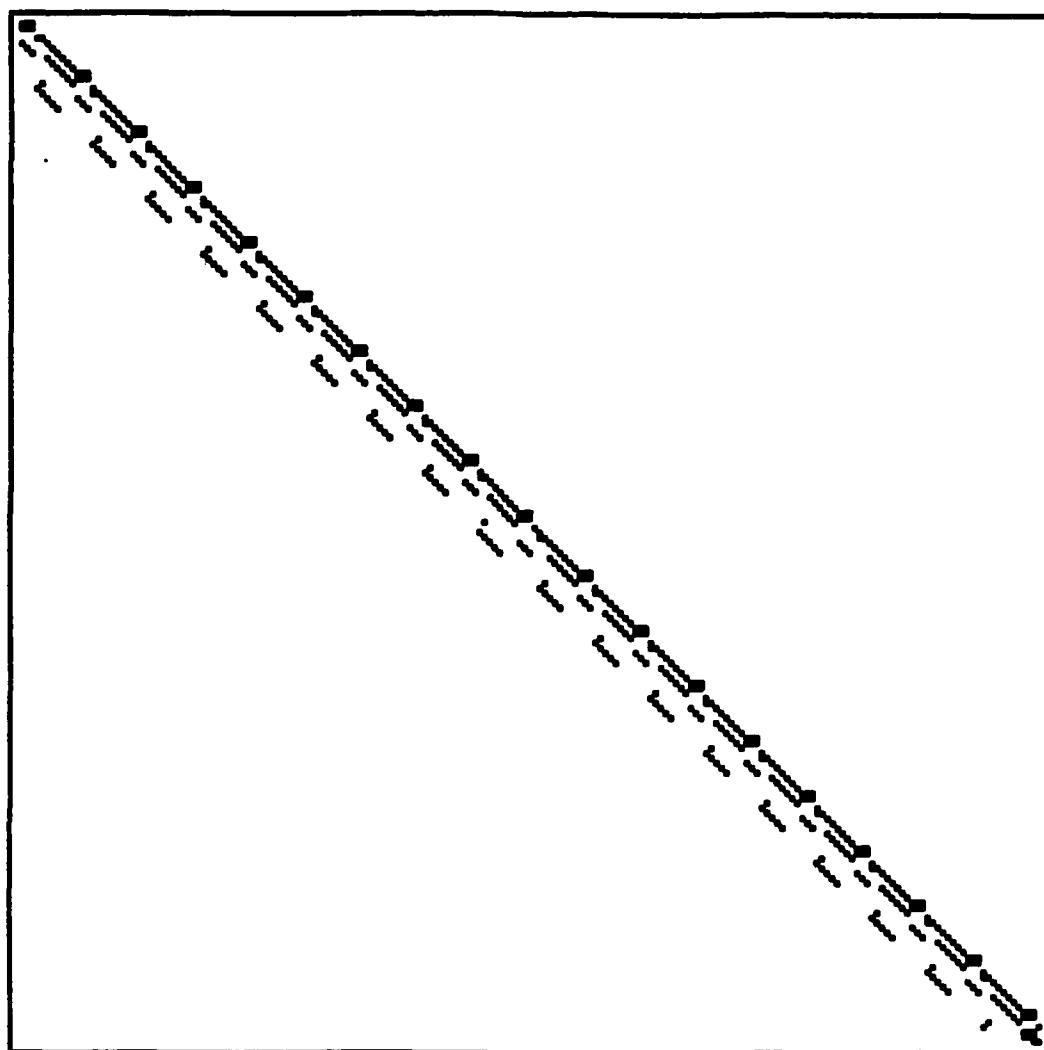




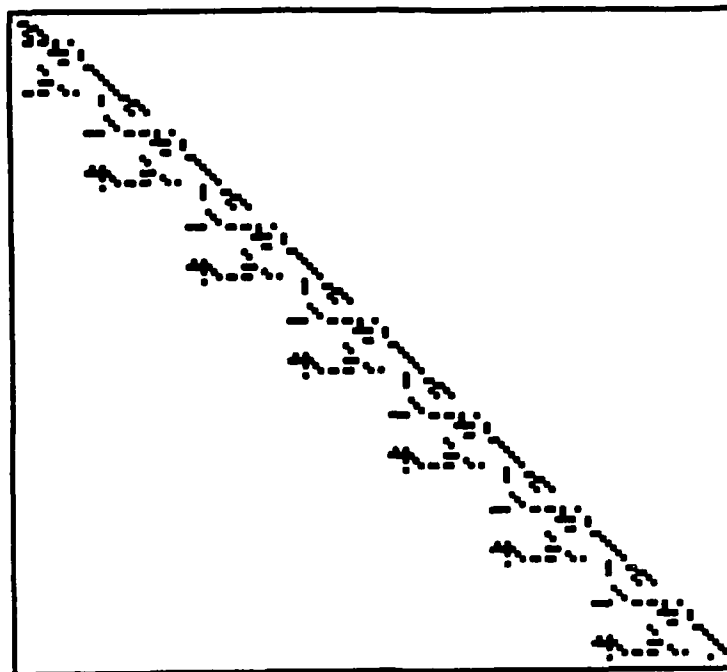
Picture for Problem 1 AFIRO Magnified 10.0



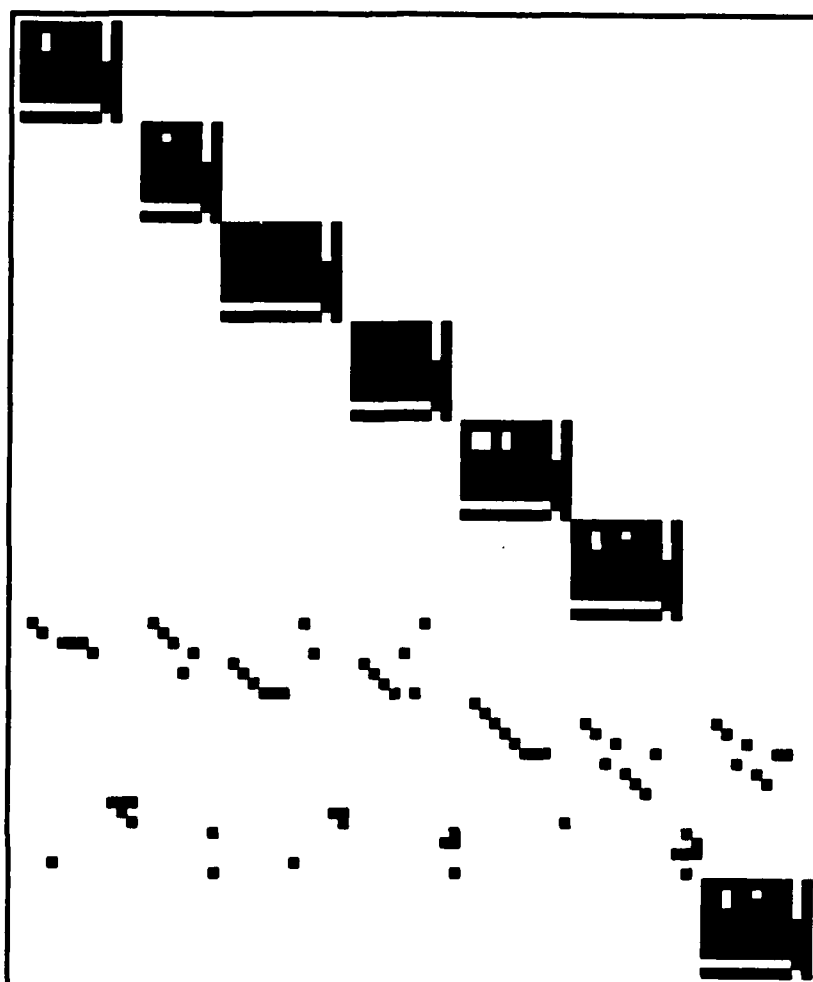
Picture for Problem 2 ADLITTLE Magnified 4.0



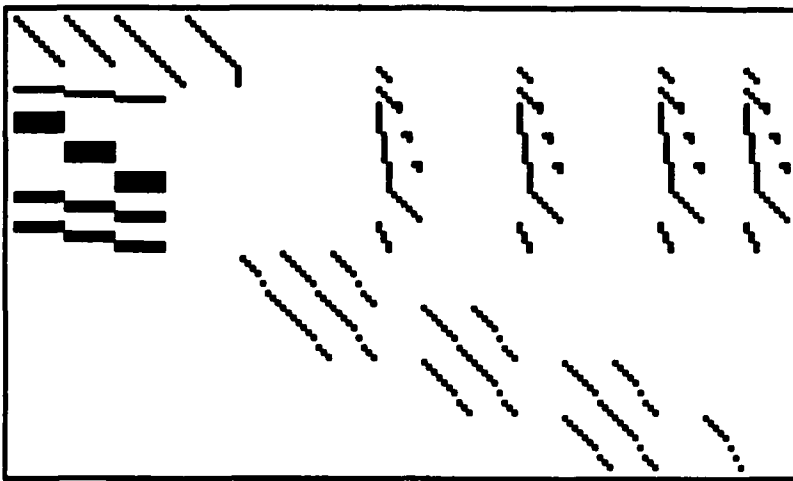
Picture for Problem 3 SC205 Magnified 2.0



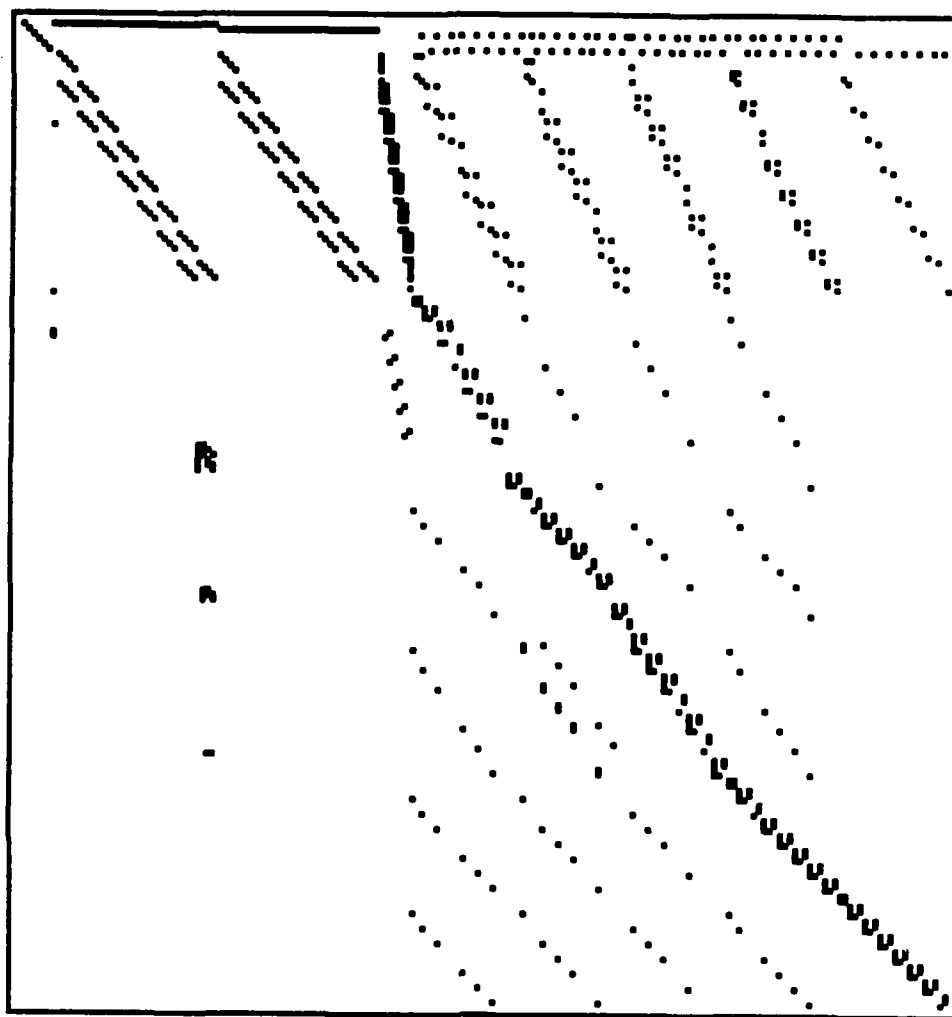
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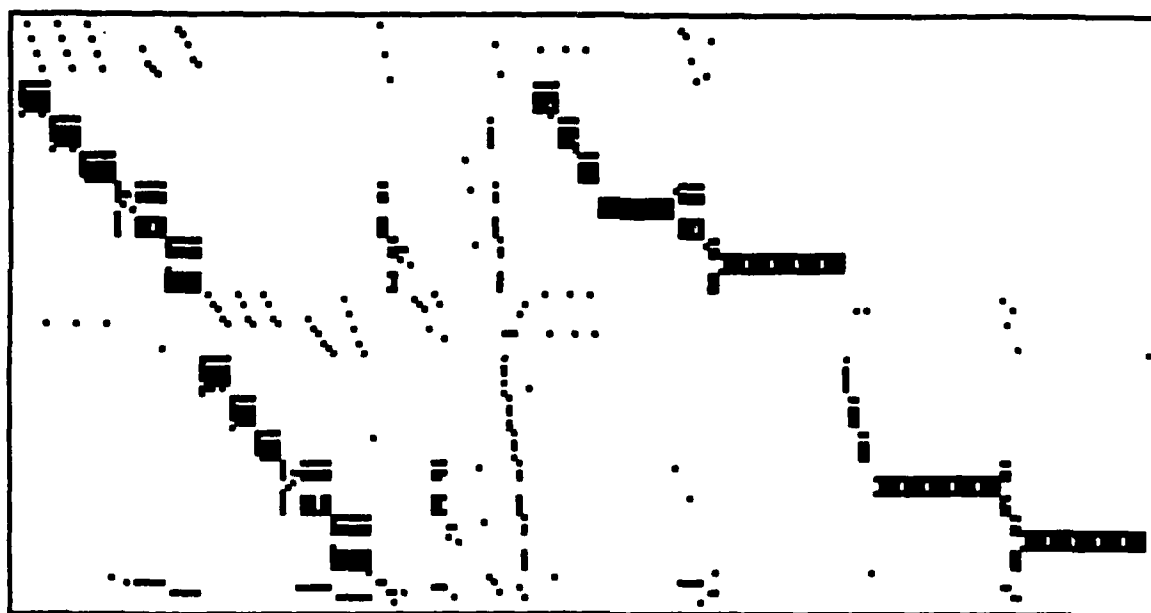
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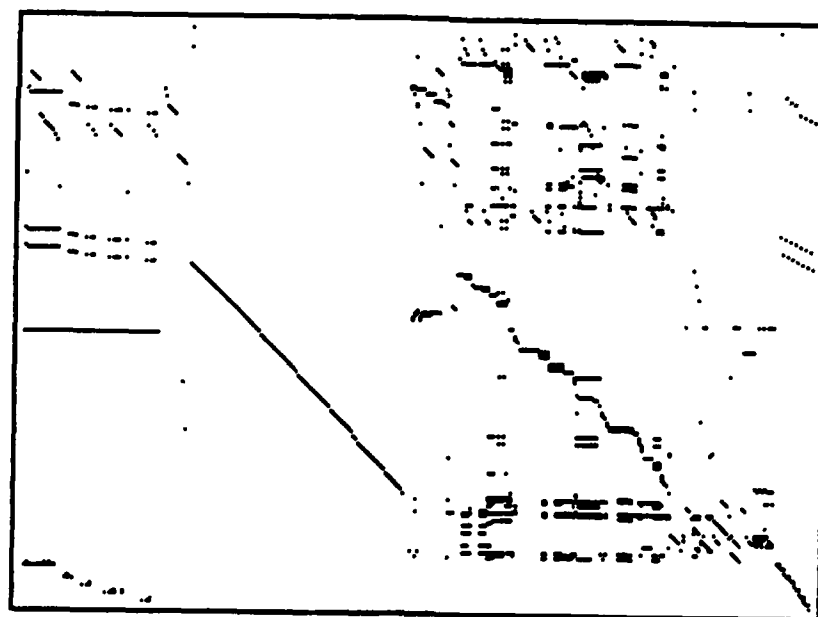
Picture for Problem 6 RECIPE Magnified 2.0



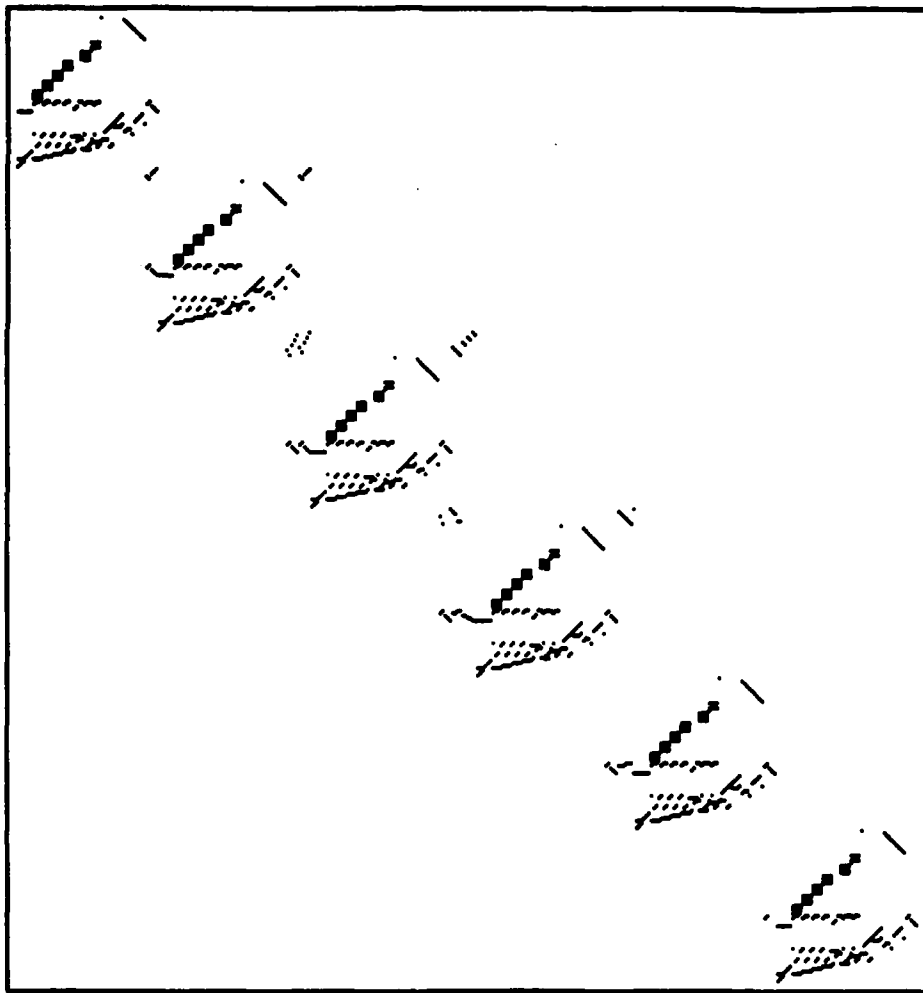
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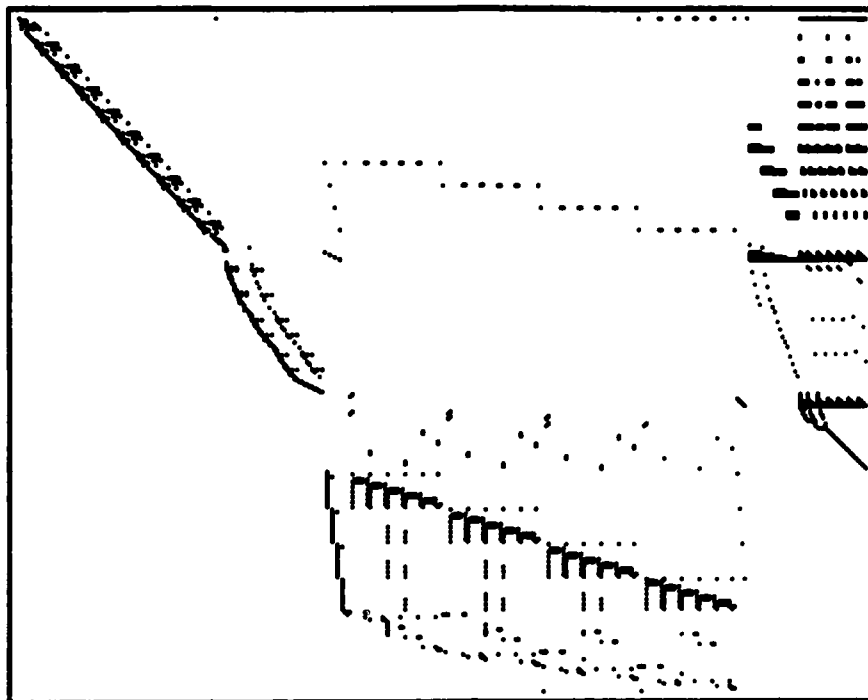
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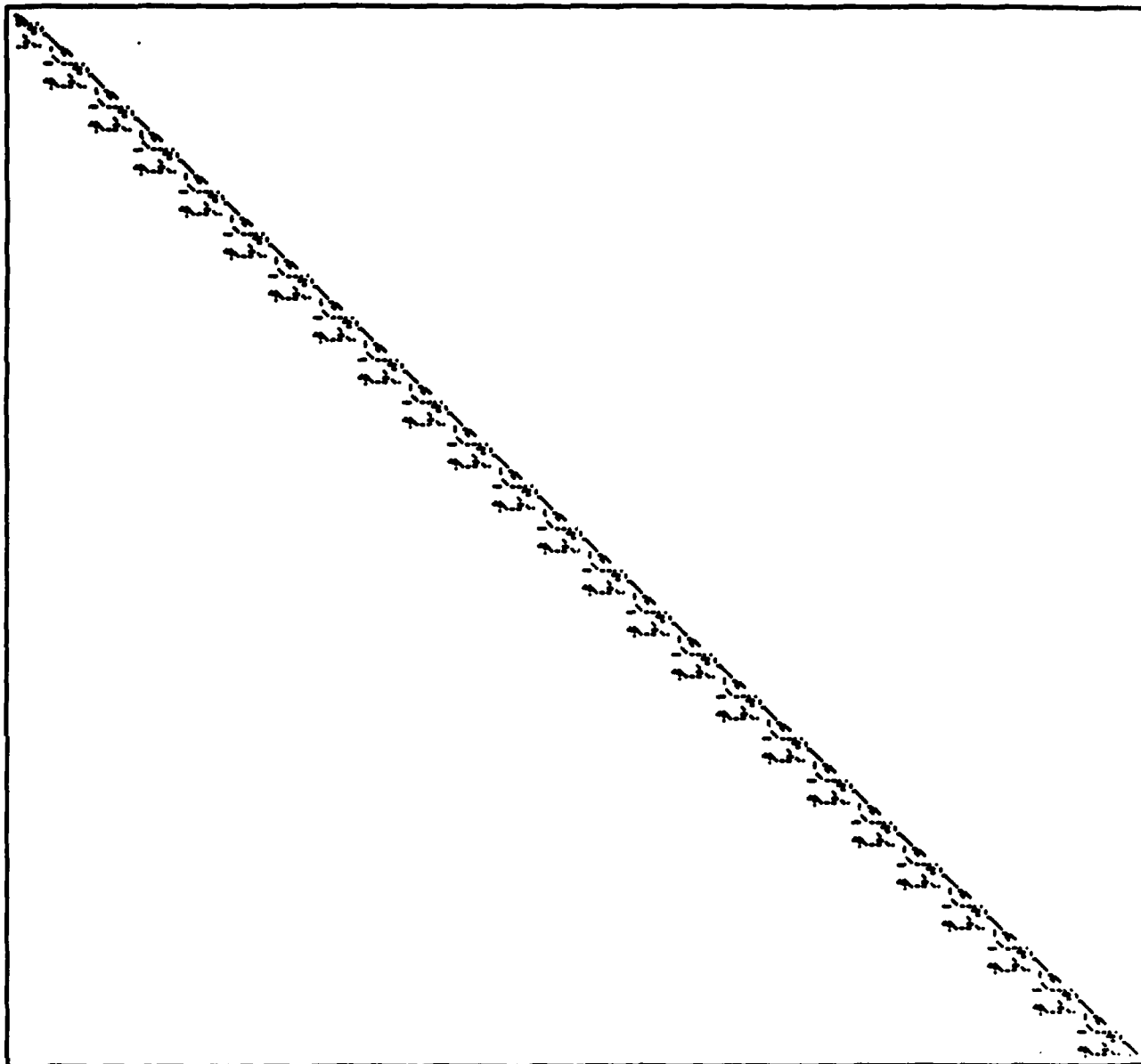
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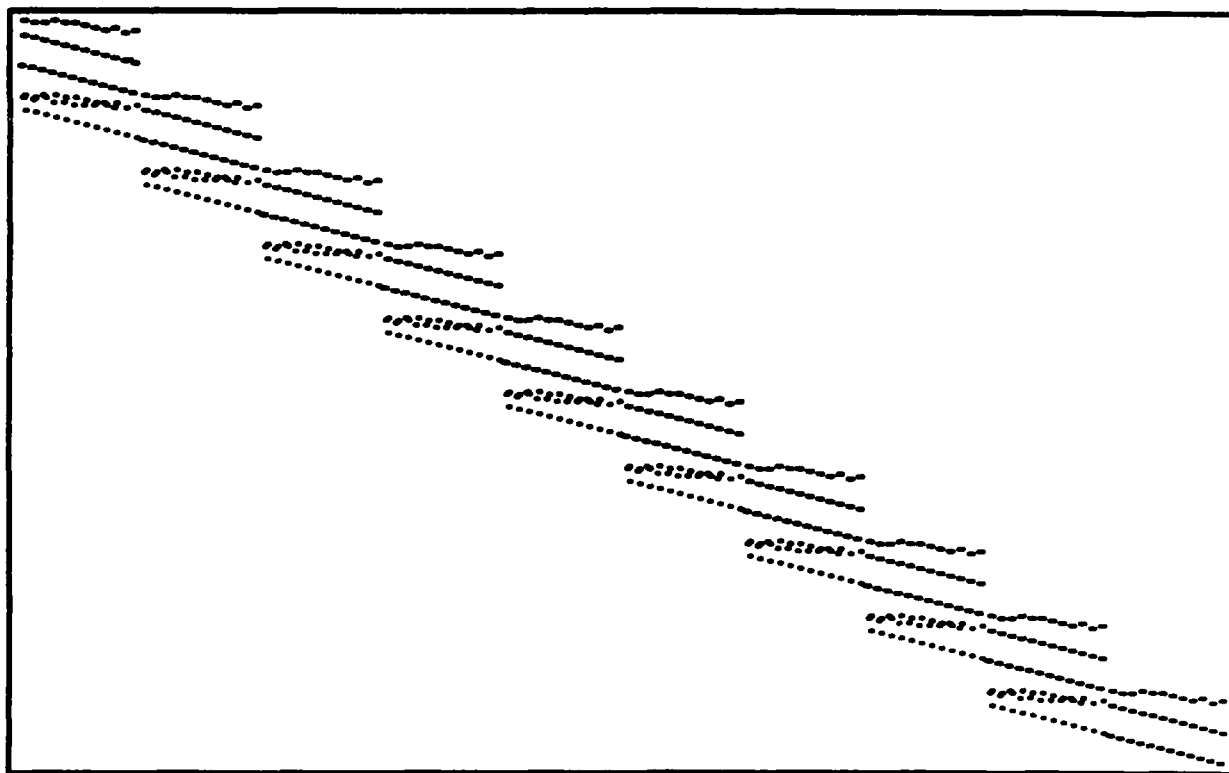
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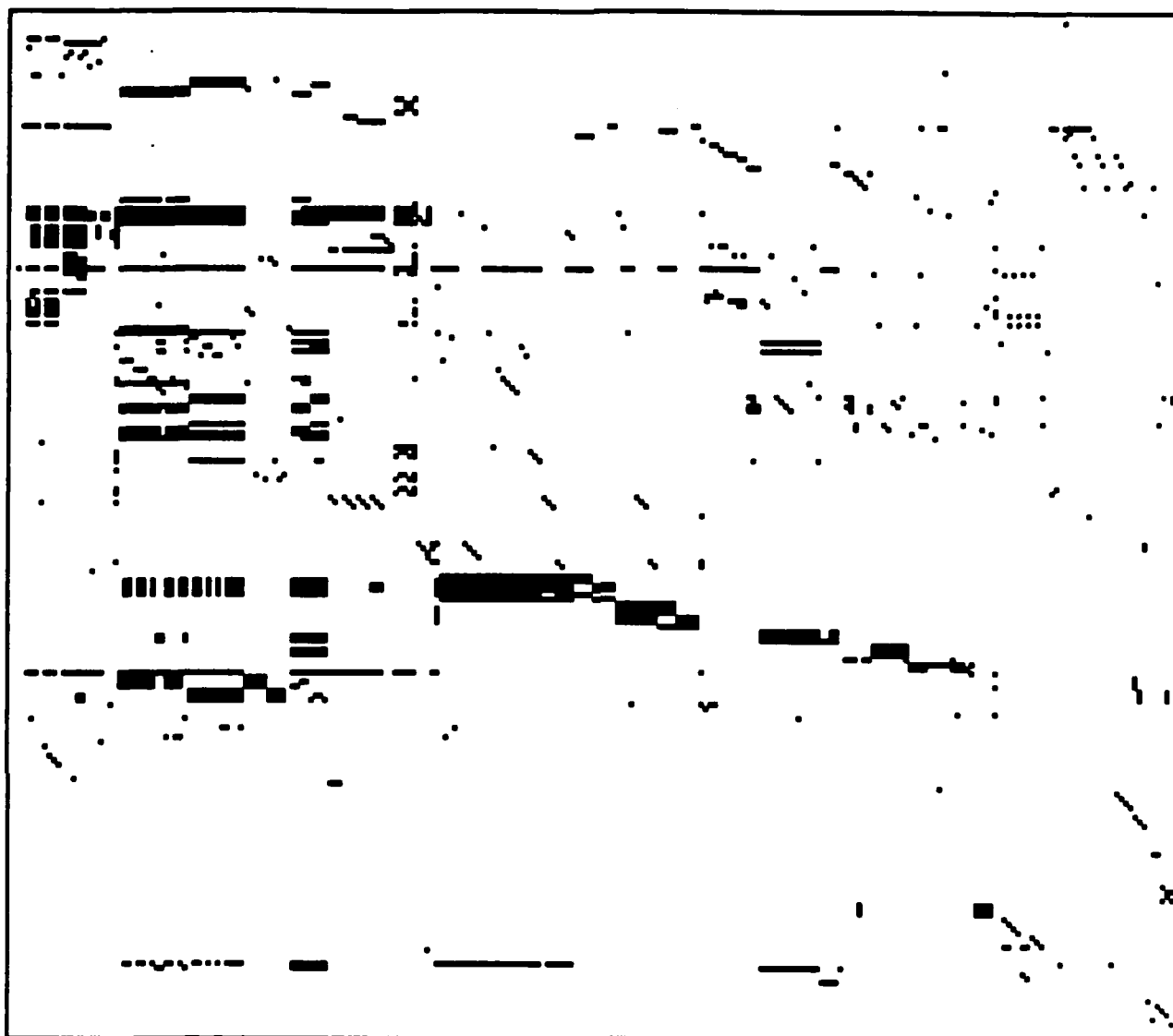
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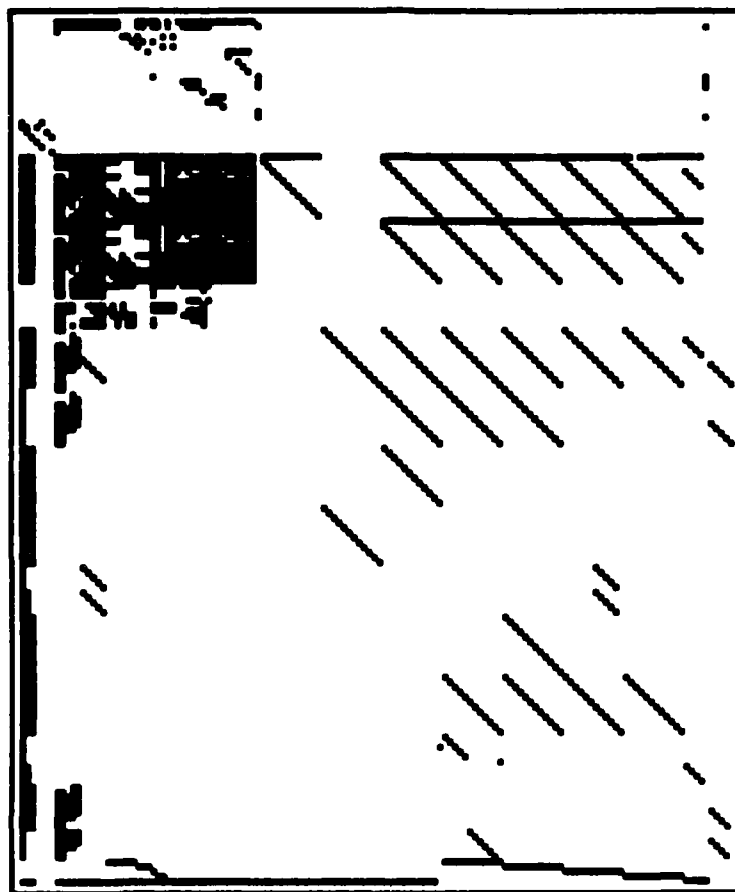
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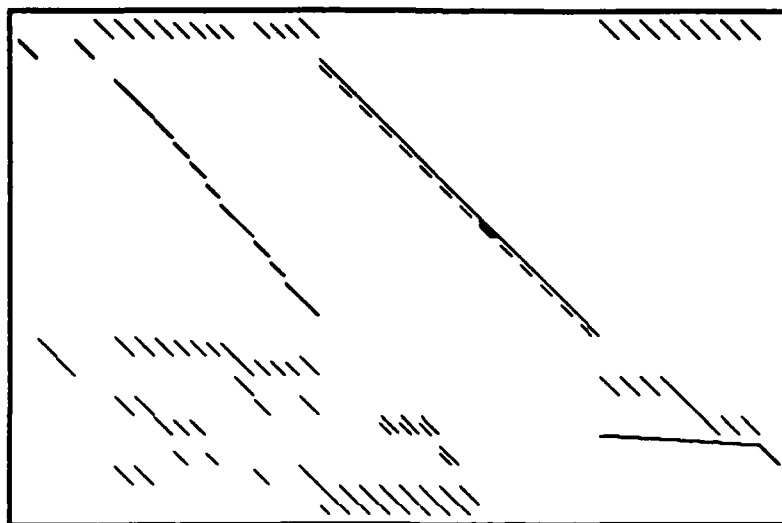
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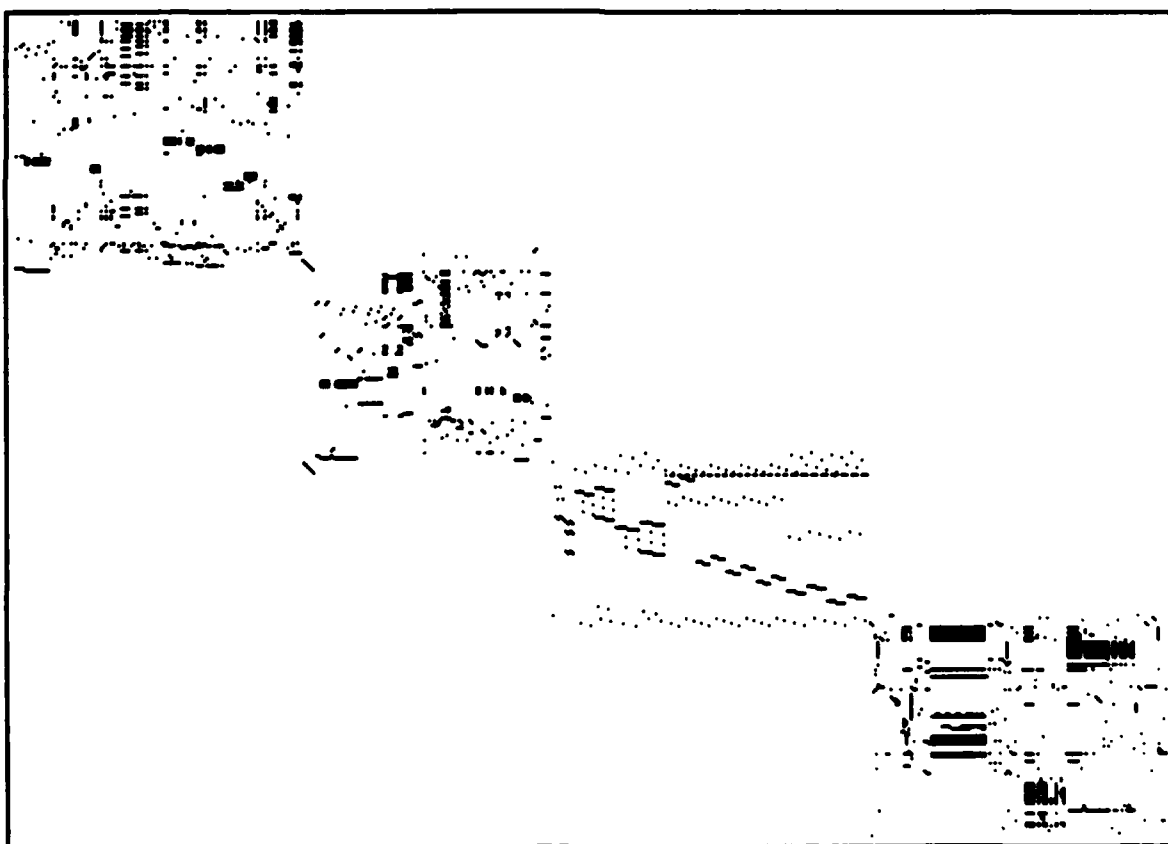
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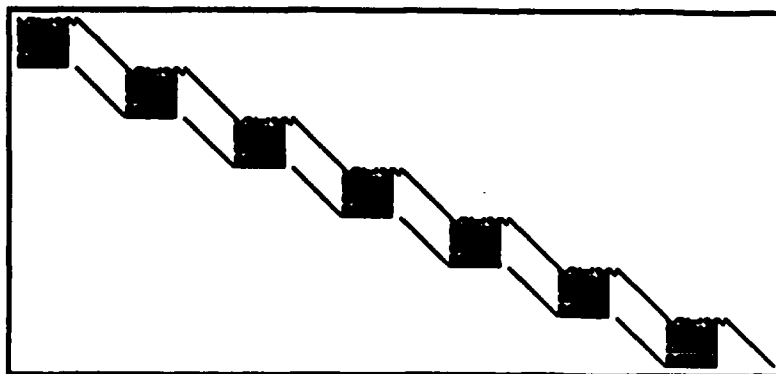
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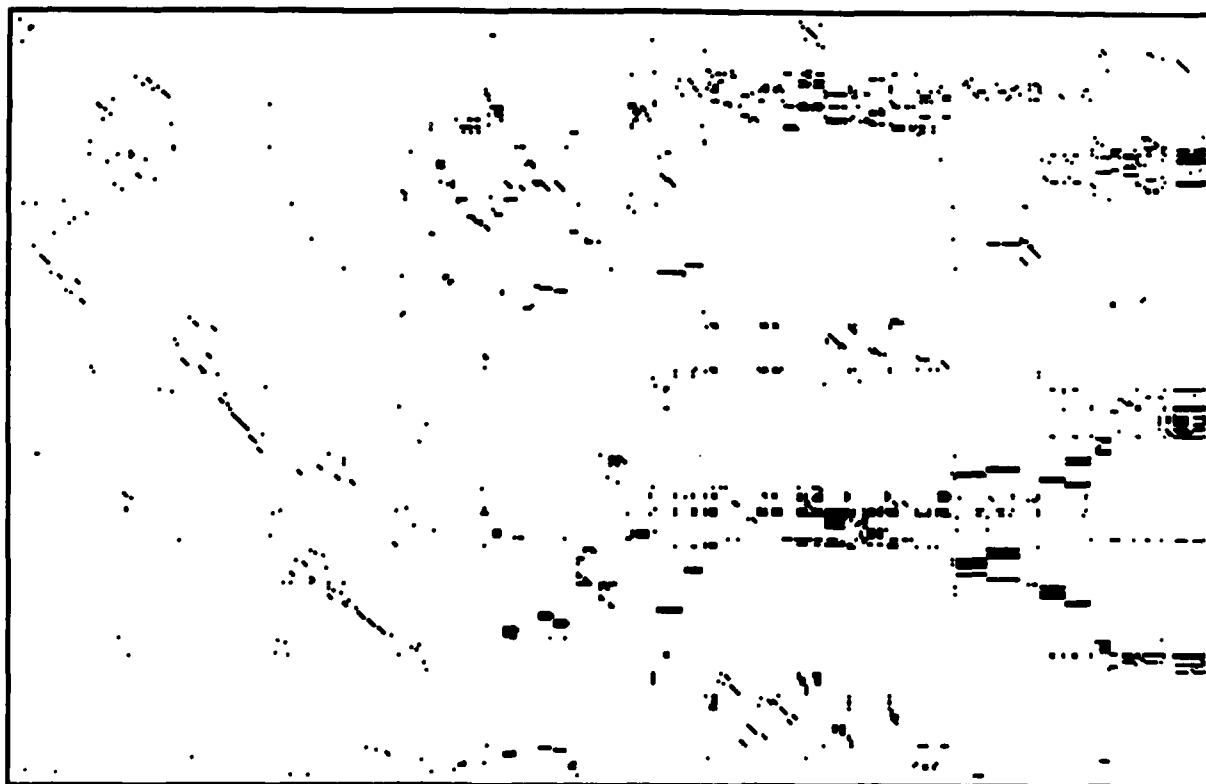
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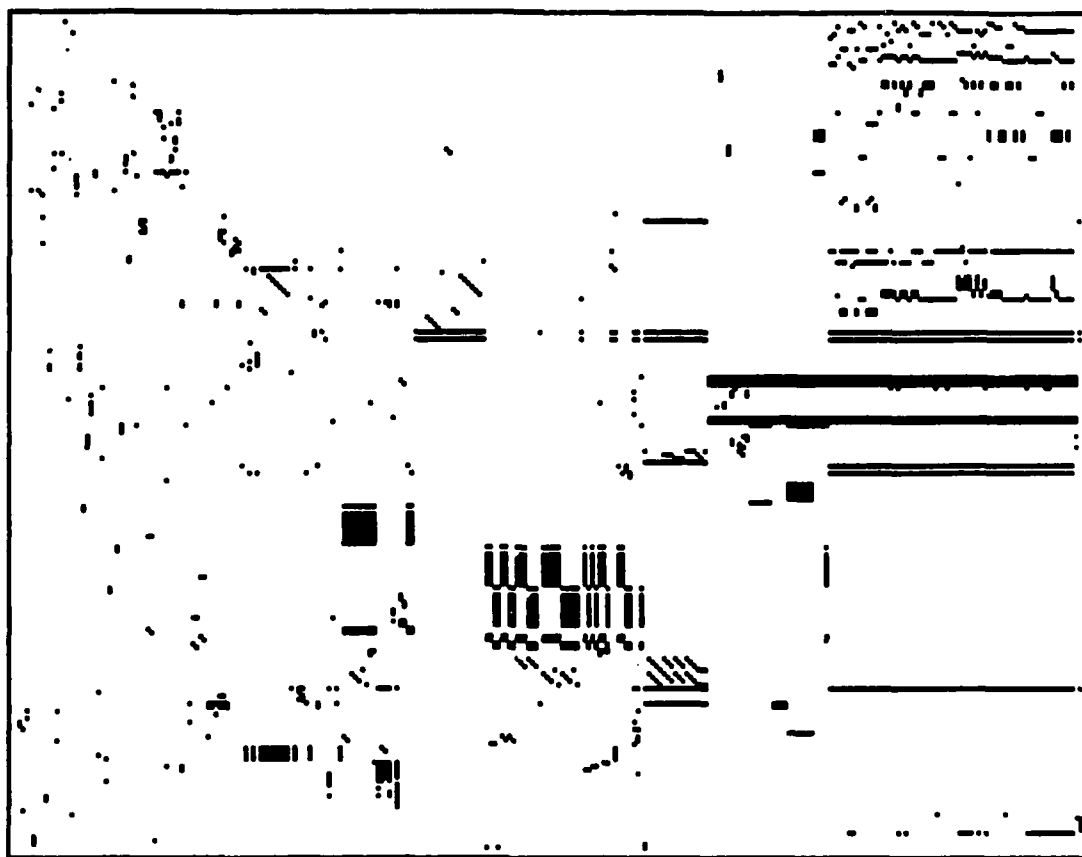
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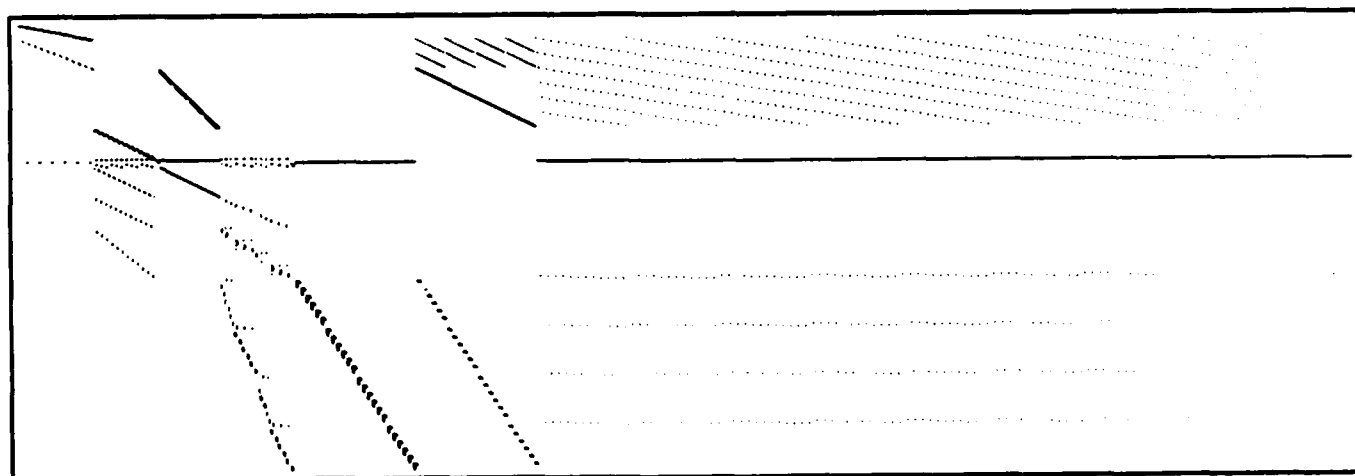
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Picture for Problem 19 BANDM Magnified 1.0



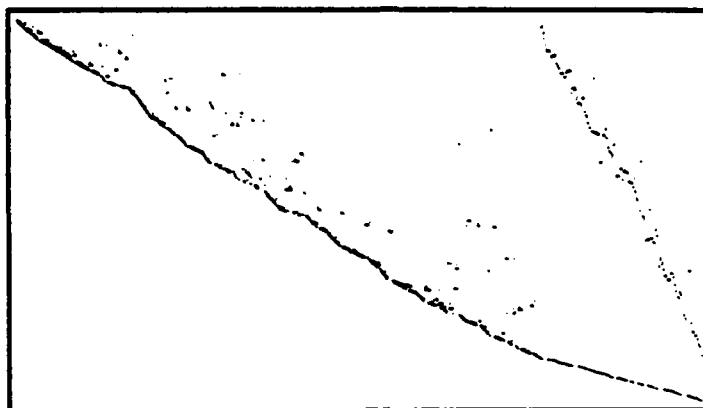
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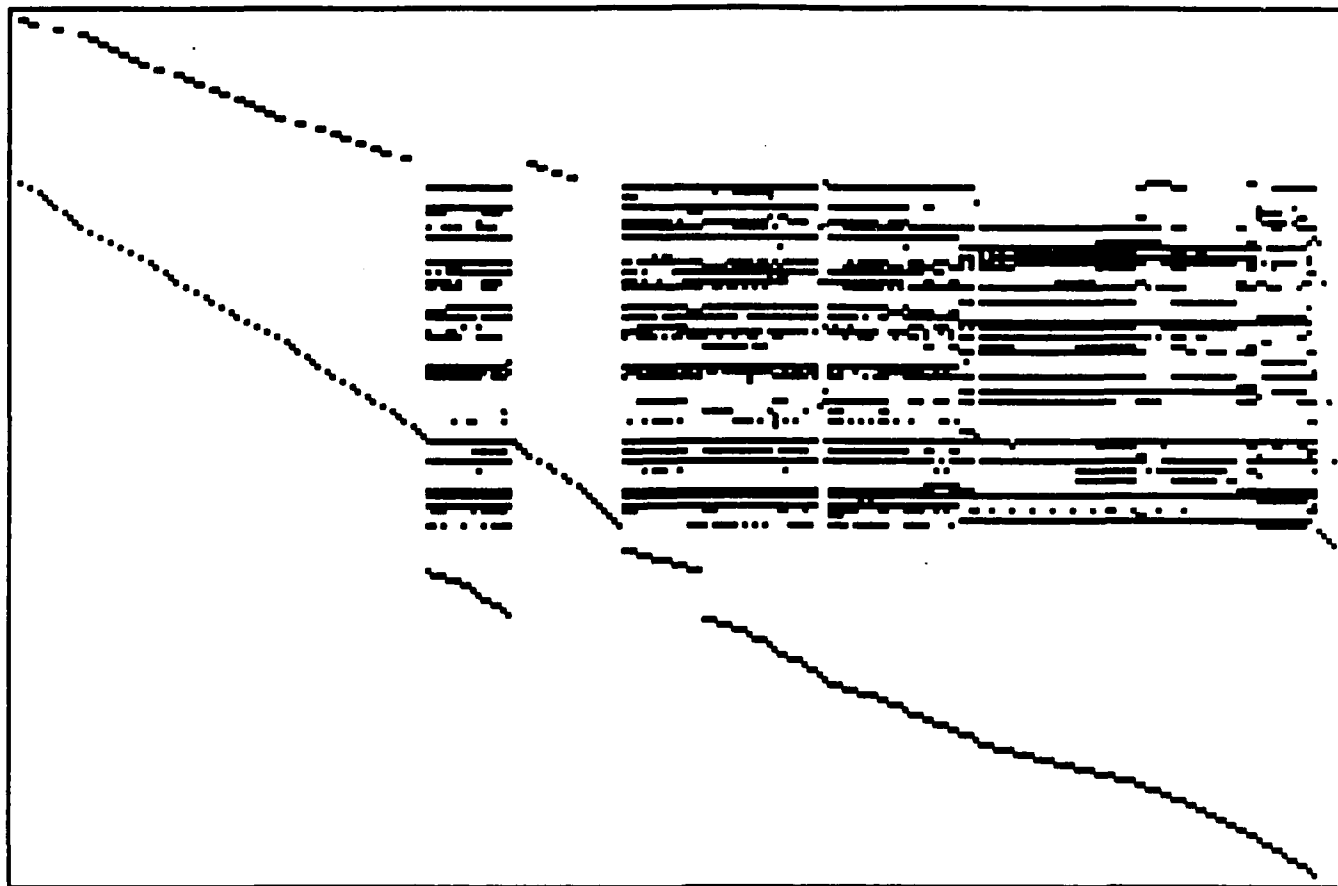
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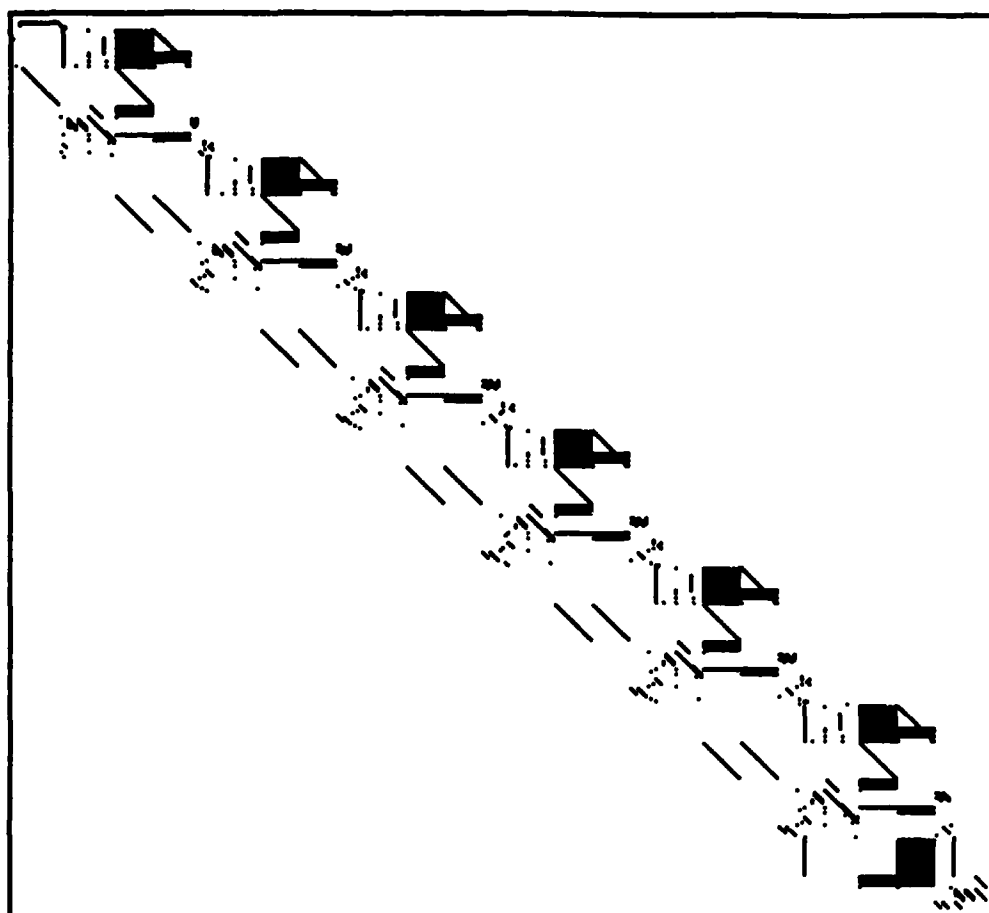
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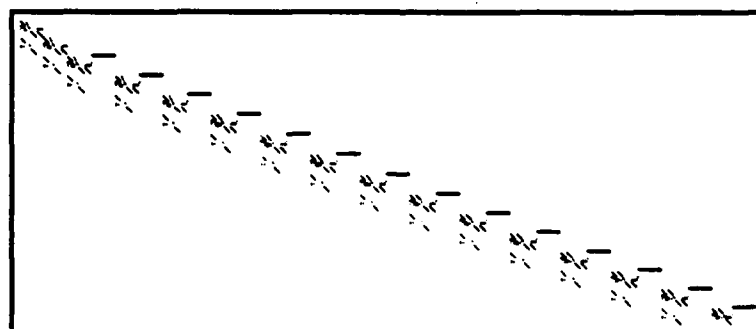
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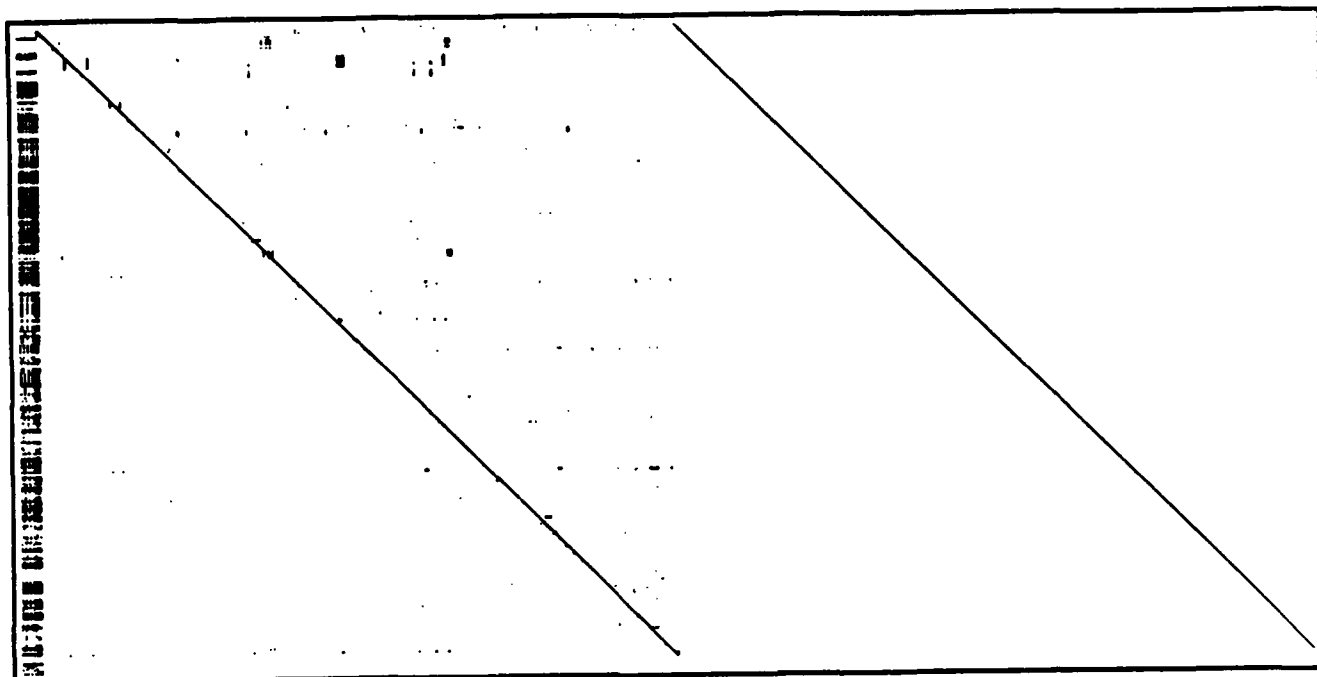
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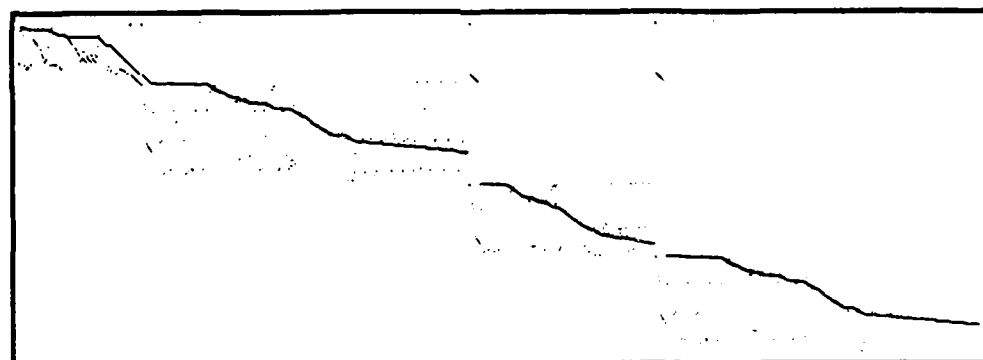
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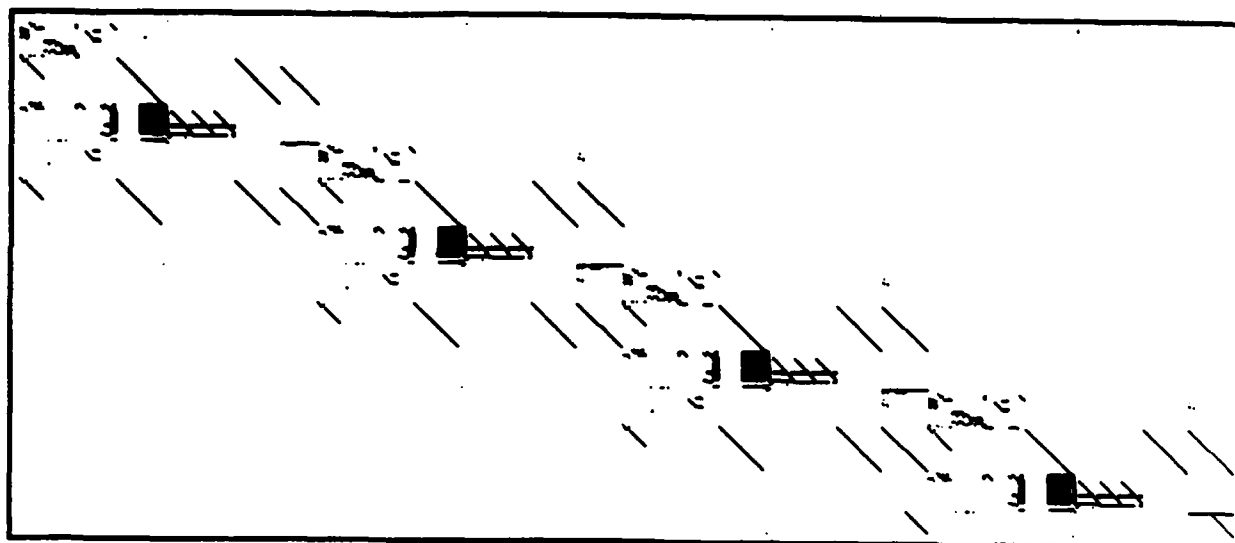
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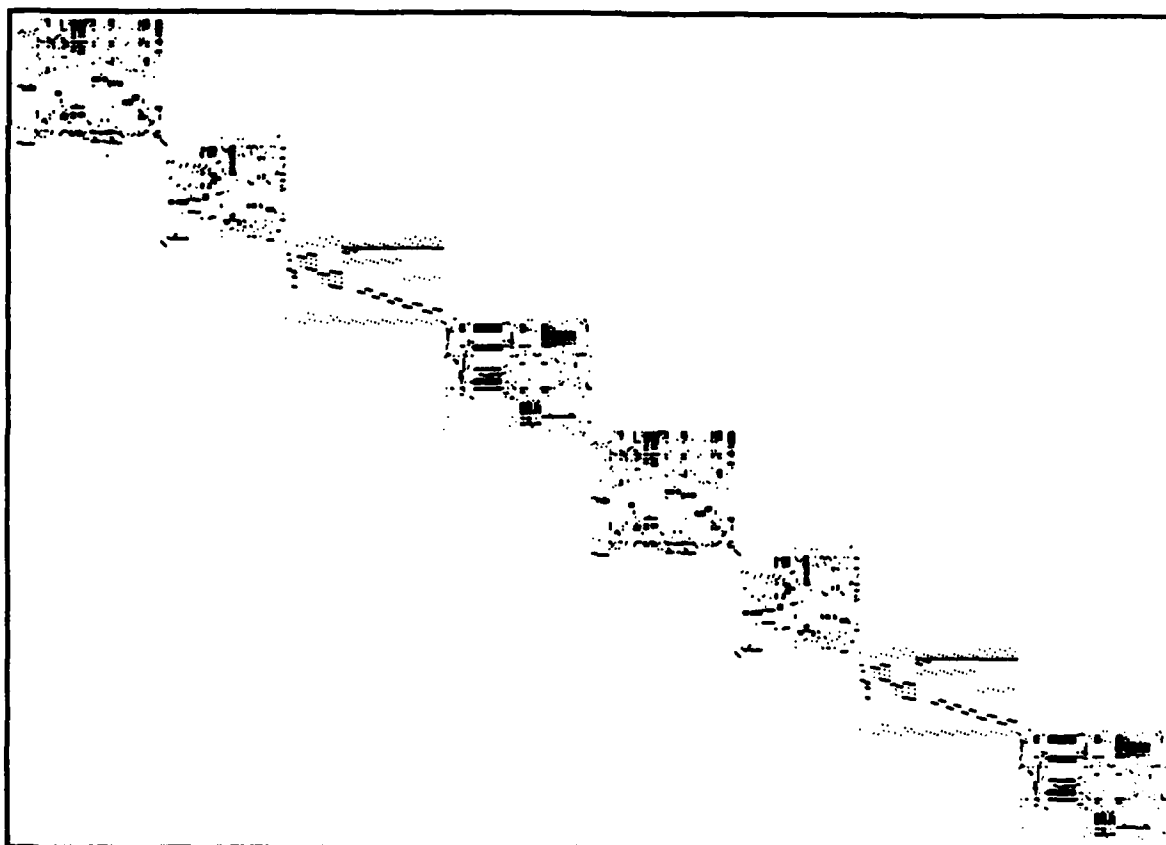
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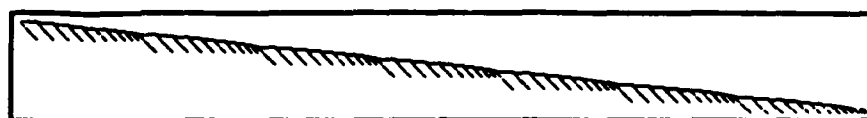
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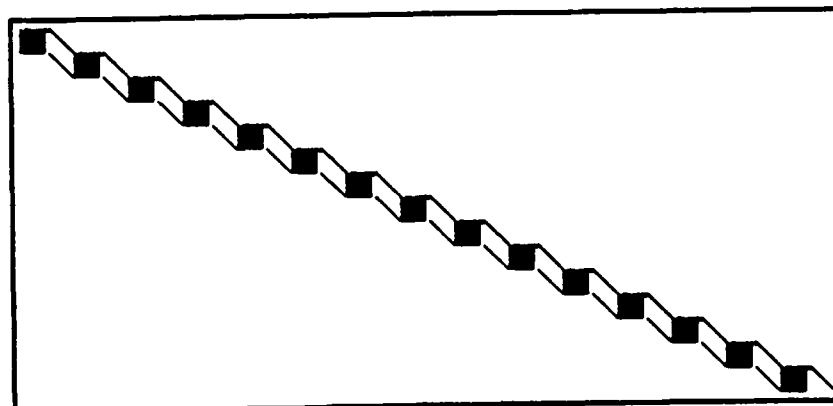
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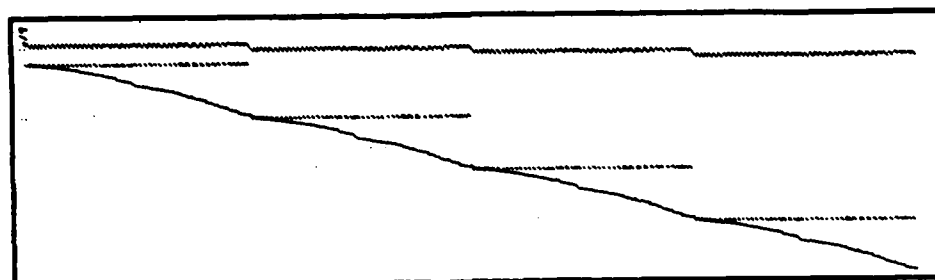
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Picture for Problem 31 SCSD6 Magnified 0.25



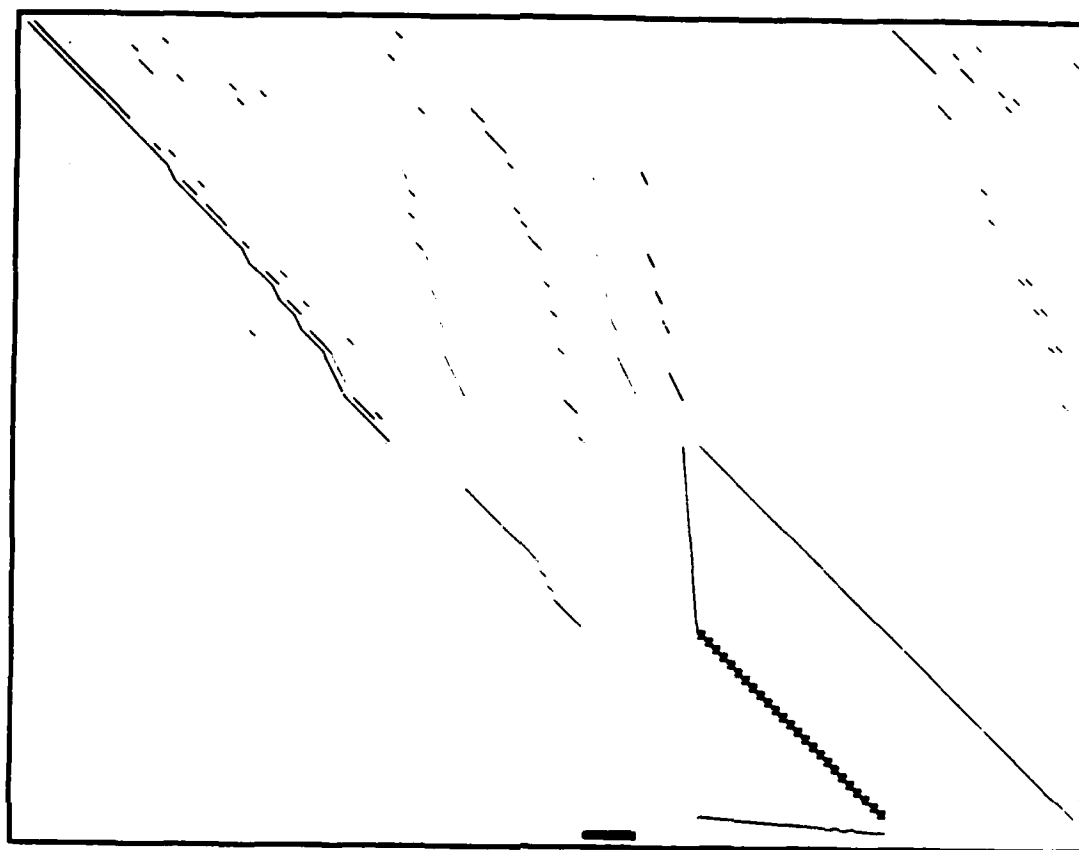
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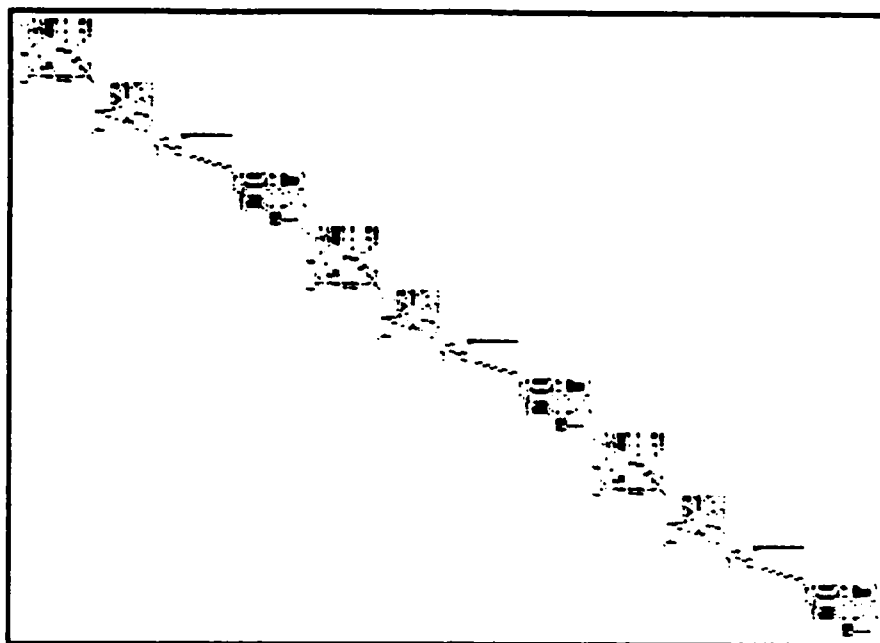
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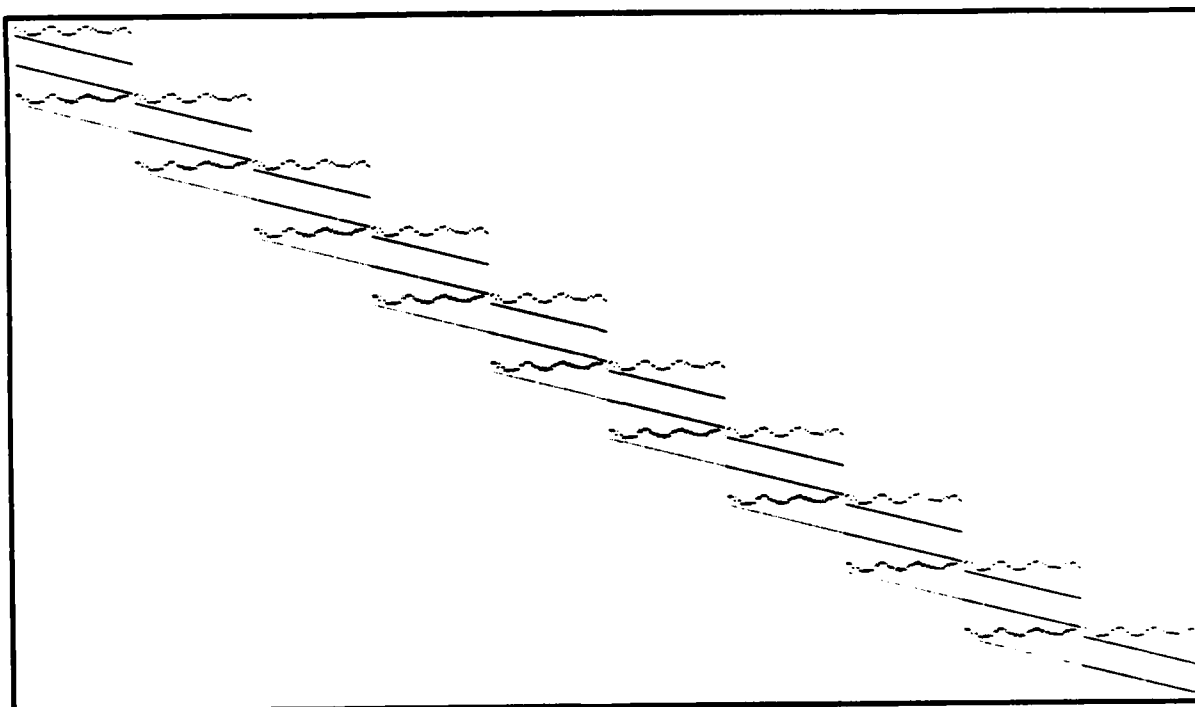
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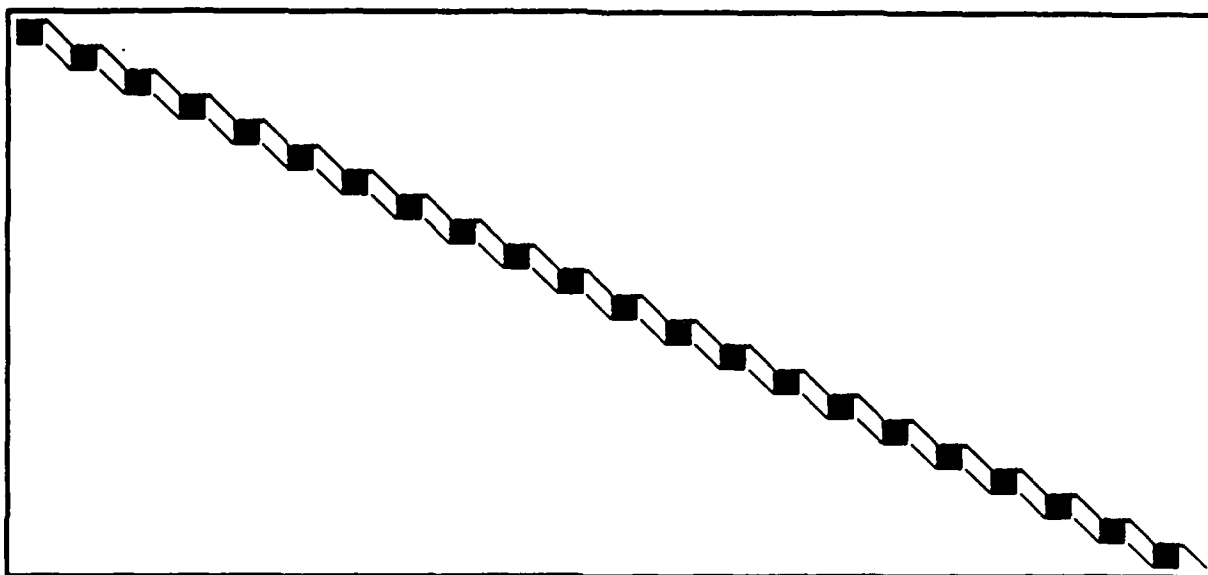
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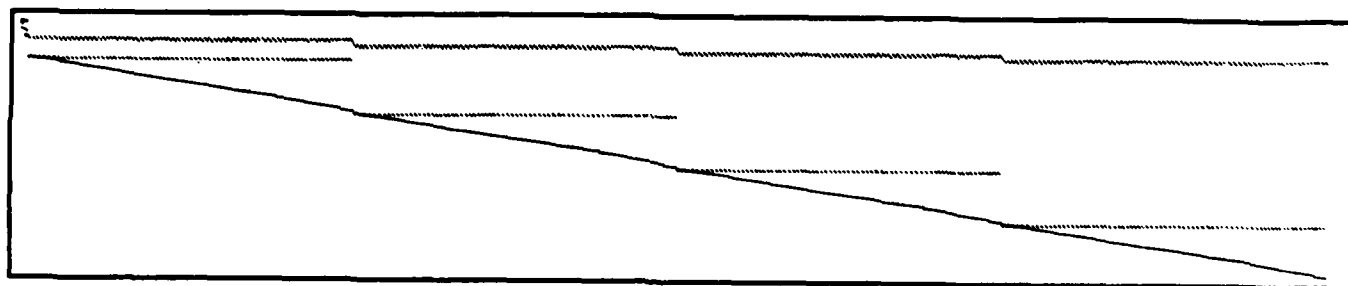
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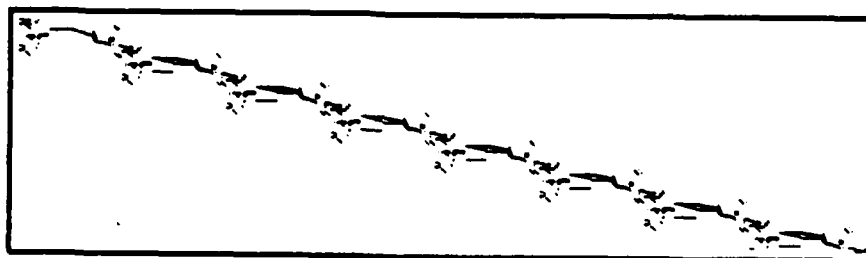
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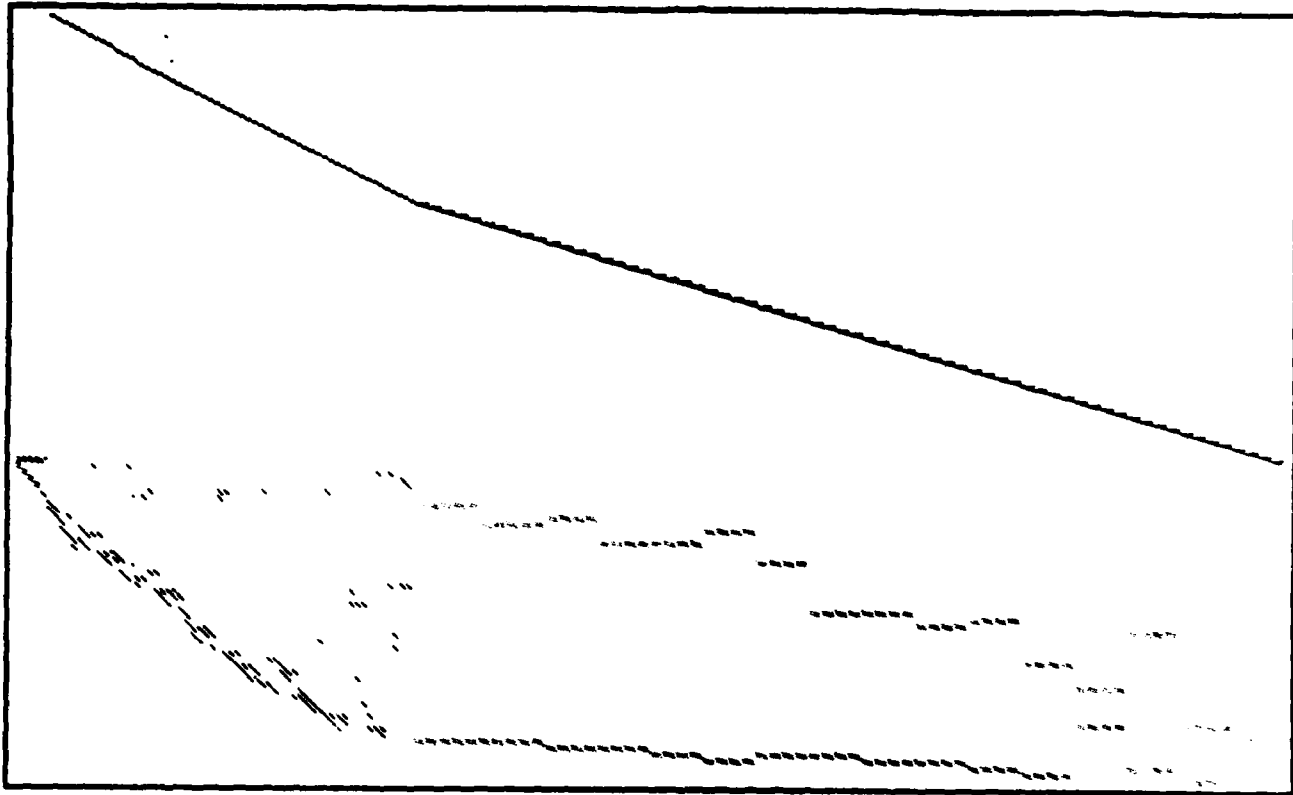
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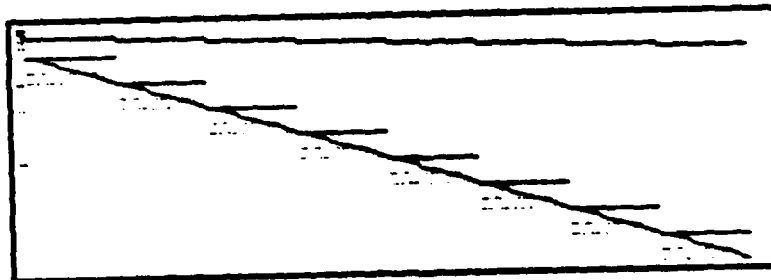
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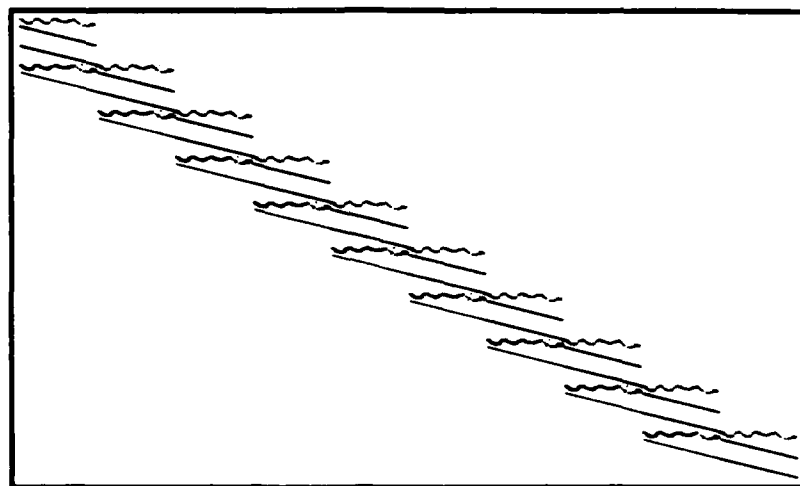
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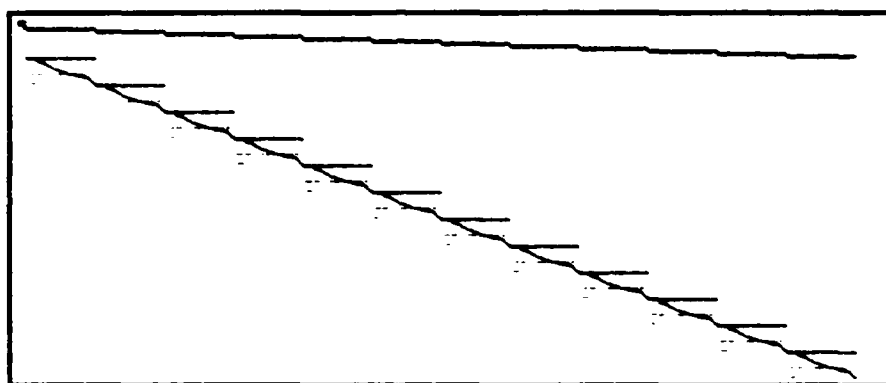
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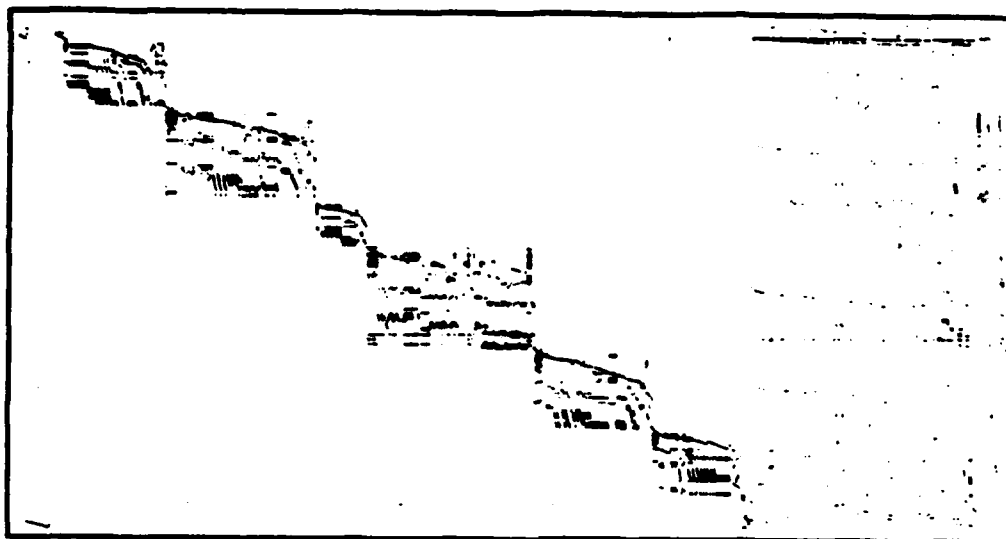
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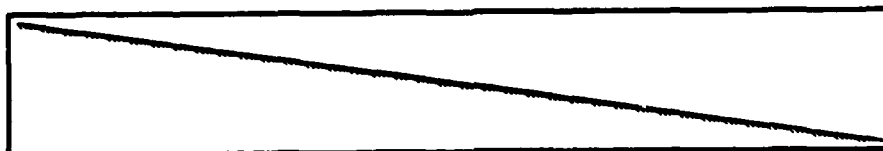
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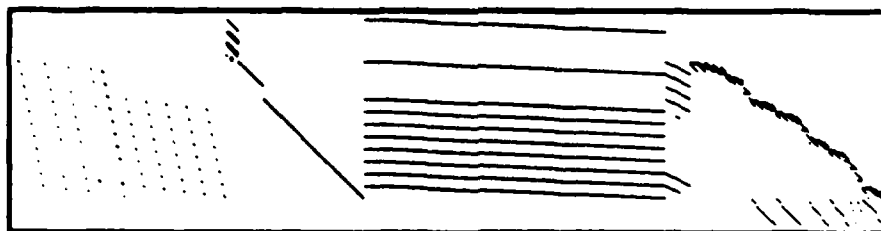
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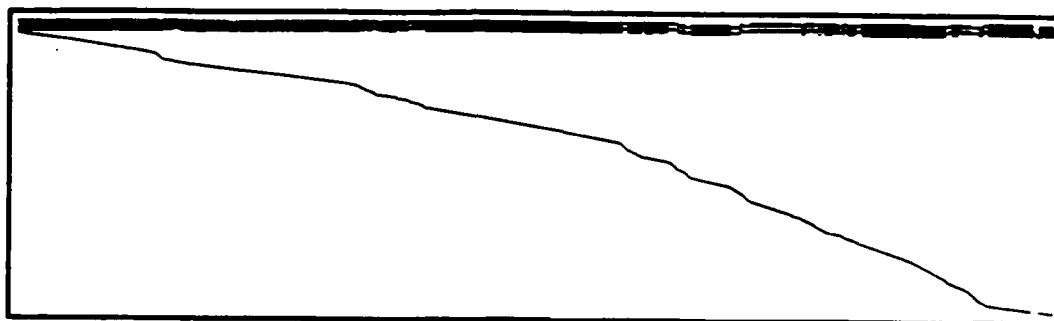
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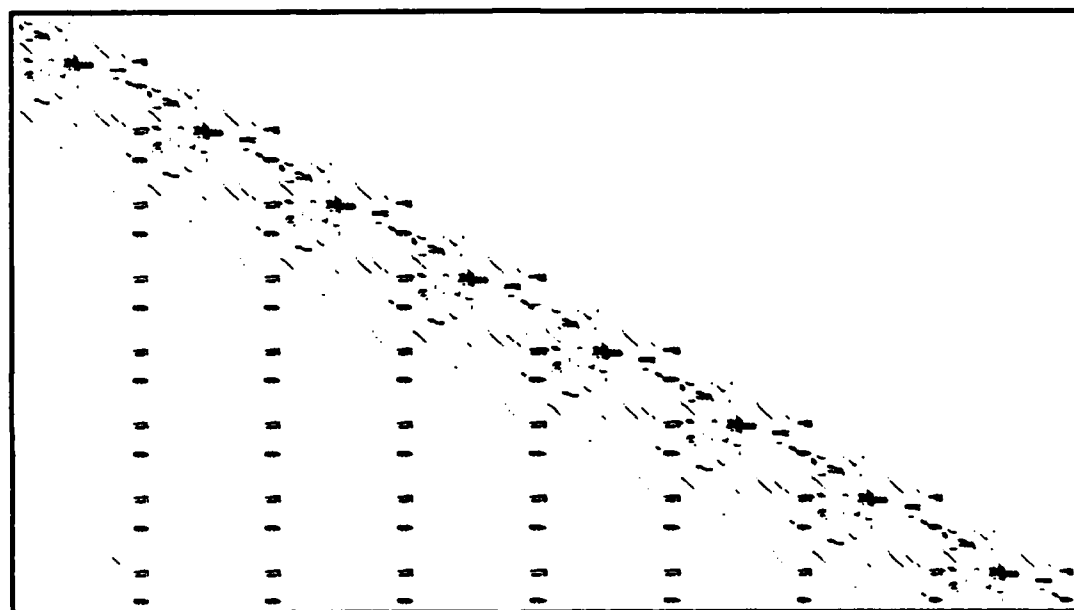
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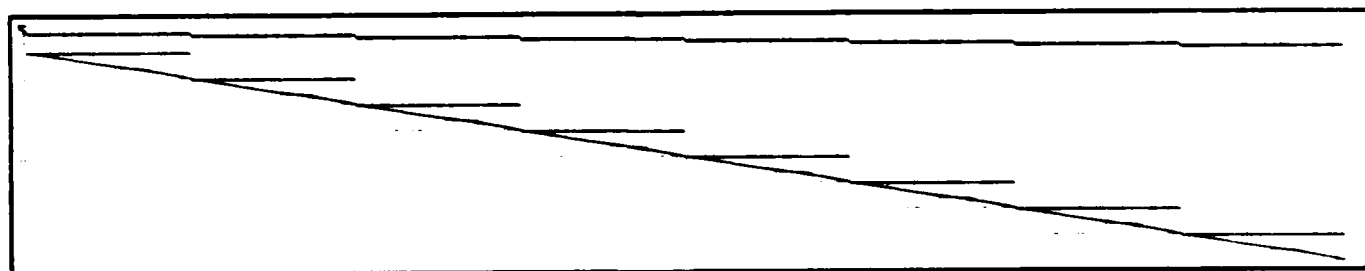
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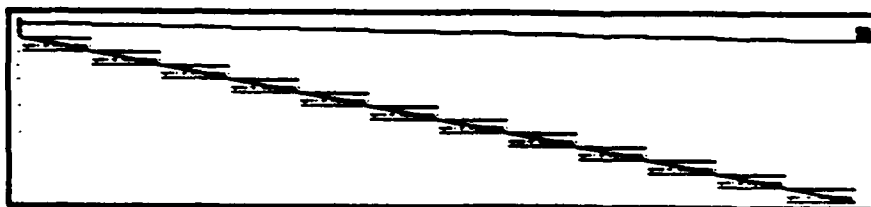
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Picture for Problem 49 PILOTJA Magnified 0.25



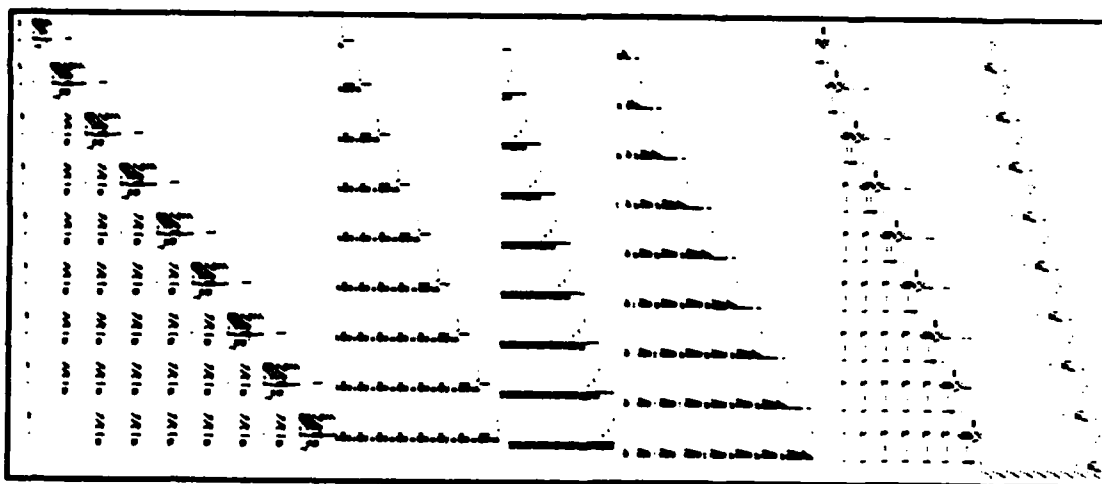
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Picture for Problem 51 SHIP12L Magnified 0.0625



Picture for Problem 52 80BAU3B Magnified 0.05



Picture for Problem 53 PILOT Magnified 0.125

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Abstract

A set of linear programming test problems is analyzed with MINOS, Version 5.1. The problems have been run with different options for scaling and partial pricing to illustrate the effects of these options on the performance of the simplex method. The results indicate that the different options can significantly improve or degrade the performance of the simplex method, and that these options must be chosen wisely.

For each problem, a picture of the nonzero structure of the matrix A is also presented so that the problems can be classified according to structure.

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